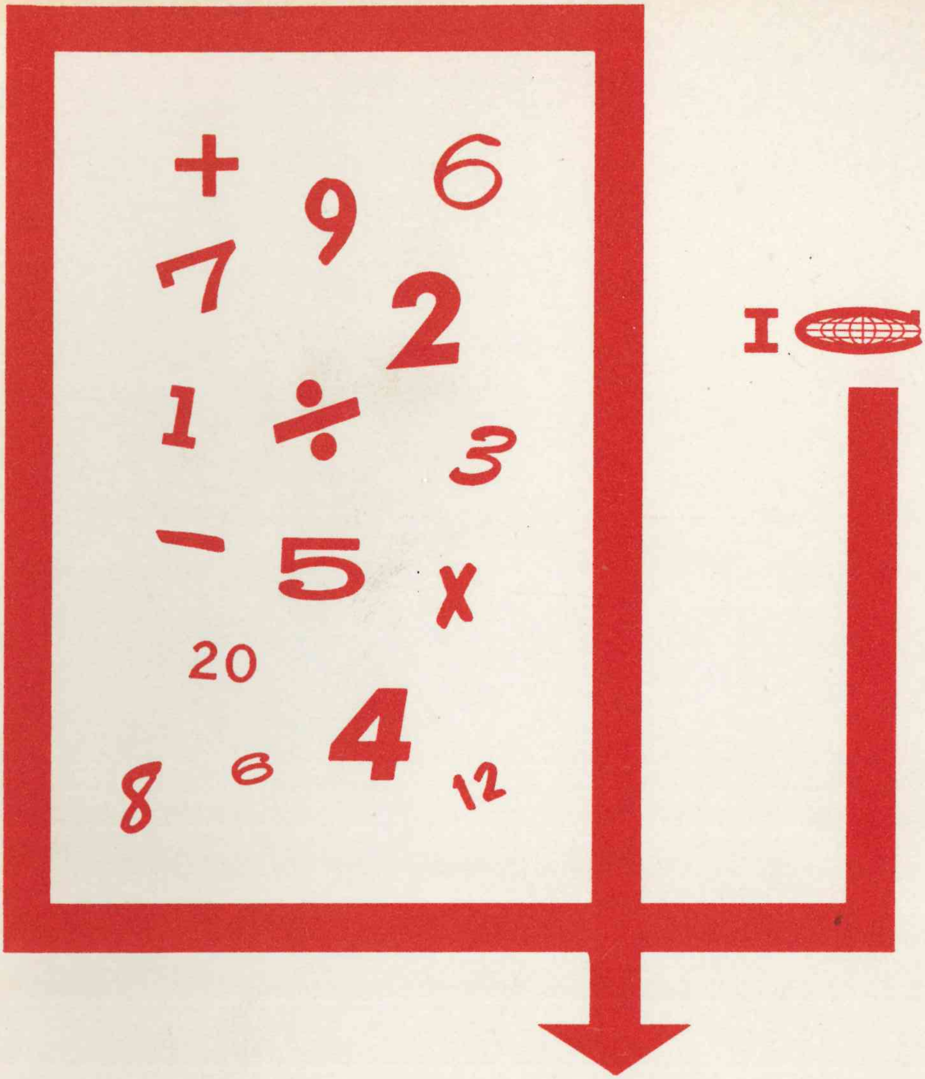

ICS

Practical Arithmetic

Fractions

By

W. M. SPANGLER, M. A.

6750B
Part 2
Edition 1

Practical Arithmetic
PART 2

Fractions

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What This Text Covers . . .

There is hardly a day in the life of the average man when he doesn't use fractions in his work. Here is your chance to learn how to use them properly. This text shows you how the four basic operations of arithmetic apply to fractions. Follow the outline below for easy studying.

1. What Is a Fraction? . . . . Pages 1 to 11

   Here you'll learn what a fraction is and how important fractions are in everyday life. You'll also learn the correct methods of writing fractions, and you'll find out all about mixed numbers.

2. Reducing Fractions . . . . Pages 12 to 22

   In this section you'll learn how to change the form of a fraction without changing its value. And you'll see how mixed numbers and integers are changed to fractions.

3. How to Add Fractions . . . . Pages 23 to 27

   Here you'll learn what a common denominator and a least common denominator are and how to find them. When you've mastered this, the rest is easy. Just apply what you already know about addition.

4. How to Subtract Fractions . . . . Pages 28 to 32

   Subtraction is simply the reverse of addition. If you can add fractions, you'll have no trouble subtracting them.

5. How to Multiply Fractions . . . . Pages 33 to 39

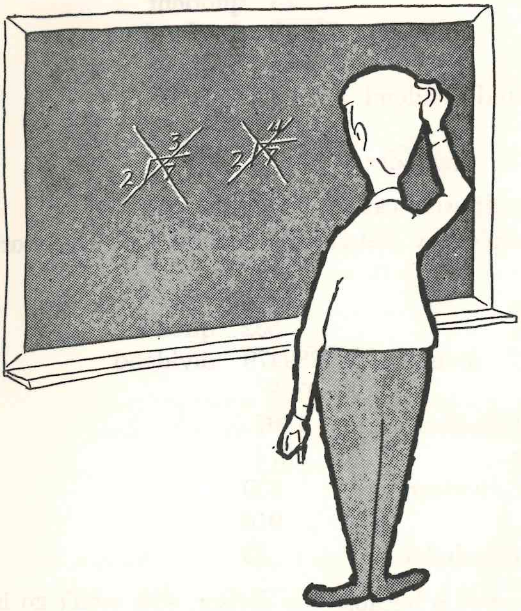
   Multiplying fractions is actually easier than adding or subtracting them. Once you learn the standard method of multiplication, you'll be introduced to cancellation, a shortcut used to simplify the multiplication process.

6. How to Divide Fractions . . . . Pages 40 to 44

   To divide fractions you just invert the divisor and multiply. Memorize this rule. Here you'll also learn what a reciprocal is and how to find it.

7. More About Fractions . . . . Pages 45 to 49

   The fractions you've dealt with so far have been relatively simple. Now you're ready for more complex fractions. The order of operations, a very important principle, is also discussed here. Learn it well.



Practical Arithmetic

Part 2

Fractions

Facts You Should Know About Fractions

Why Fractions?

1. So far in your study of arithmetic, you've dealt with whole numbers only. In fact, up to now, you've had no need for any number other than a whole number. And as long as you do only addition, subtraction, and multiplication, you won't meet any problem that requires any new kind of number. In other words, you can add whole numbers, subtract whole numbers, and multiply whole numbers for just as long as you like, and each and every time the result will be a whole number. However, you won't do many division problems before you discover that there are times when a whole number is not and cannot be the complete answer. In a division problem as simple as 163 ÷ 7, for example, you'll find that the quotient is not 23 nor is it 24.

Let's take a closer look at this basic operation, division, which requires us to enlarge our ideas and concepts of arithmetic. Doing the actual division of 163 by 7 as you learned in the preceding text, you'll get this:

$$\begin{array}{rrl}
 & 23 & \text{quotient} \\
\text{divisor} & 7\overline{)163} & \text{dividend} \\
 & \underline{14} & \\
\text{partial dividend} & 23 & \\
 & \underline{21} & \\
\text{remainder} & 2 &
\end{array}$$

Since 7 won't go into 2, the division cannot be continued.

Let's take another example, say 289,479 divided by 458. The work will look like this:

$$\begin{array}{rrl}
 & 632 & \text{quotient} \\
\text{divisor} & 458\overline{)289479} & \text{dividend} \\
 & \underline{2748} & \\
\text{partial dividend} & 1467 & \\
 & \underline{1374} & \\
\text{partial dividend} & 939 & \\
 & \underline{916} & \\
\text{remainder} & 23 &
\end{array}$$

Here again the division must stop because the divisor, 458, won't go into 23.

Even though in these examples the division could not be continued, you can see that the quotient you obtained in each case is not the complete answer. If it were, multiplying the quotient by the divisor would give you the dividend. But it doesn't. If you multiply 7 by 23 you get 161, not 163; and $458 \times 632 = 289{,}456$, not 289,479. But if you take the number left when division stopped, and add it to the product of the quotient and divisor, you'll get the dividend in each case. Thus $161 + 2 = 163$, and $289{,}456 + 23 = 289{,}479$. Don't forget: The number that is left when the division must be ended is called the *remainder*.

The remainder in divisions such as these may be expressed in one of two ways. It may be expressed as a fraction and considered as part of the quotient. This is the approach covered in this text. Or it may be expressed as a decimal and annexed to the quotient. This approach will be explained in the next text, *Decimals*.

Fractional Parts

2. Now we know that integers can't meet all our needs. We need numbers other than whole numbers. These numbers can be either *fractions* or *decimals*. However, before you can know what fractions or decimals are and how to work with them, you must understand what they represent.

You know that when anything is divided or cut, the resulting parts are smaller than the whole. For example, if you cut a steel bar in half, you'll have two pieces, each smaller than the original bar. These pieces are called fractional parts of the bar. A *fractional part* is any one of the parts into which a thing is divided. This thing may be a bar, a circle, a pile of coal, a plot of ground, an inch

of material, or anything else. And there are no restrictions on either the size or the numbers of the parts into which a thing may be divided.

When the fractional parts of an object are equal, we call each part by a special name. For instance, when an object is divided into two equal parts, each part is called one-*half* of the object. If we cut an apple into three equal parts, each part is one-*third* of the apple. If the apple is divided into four, five, or six equal parts, each part would be called one-*fourth,* one-*fifth,* or one-*sixth* of the apple. Notice how the number of fractional parts into which an item is separated (say *four*) and the name of these parts (say *fourths*) are related. Because of this relation, if we know the name of the part, we can easily determine how many equal parts there are. The terms "one-half," "one-third," "one-fourth," and so on can be represented by numbers. These numbers are called *fractions.* The word "fraction" means "broken," and so fractions can be thought of as "broken whole numbers."

Fractions

3. As you've just learned, a fraction is a number that indicates what part or parts of an object are being considered. The general idea of fractions is illustrated in Fig. 1, *(a)* through *(d),* which shows circles divided into equal parts. In *(a)* a straight line divides the circle into two equal parts, one of which is shaded. Each of the two parts is one-half of the whole circle. In other words, "one-half" is

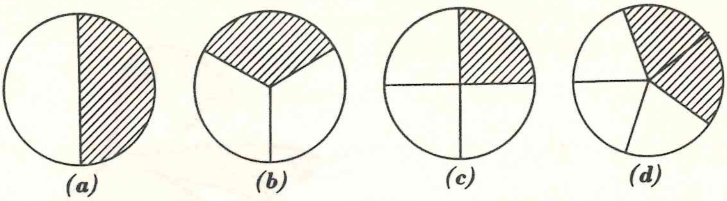


FIG. 1

a fraction, since it denotes one of two equal parts. In *(b)* the circle is divided into three equal parts, each of which is one-third of the entire circle; thus "one-third" is a fraction, because it denotes one of three equal parts. In *(c)* the circle is divided into four equal parts, each being one-fourth of the circle, and in *(d)* there are five equal parts, each being one-fifth of the circle.

Ways of Writing Fractions

4. A fraction consists of two numbers written one below the other and separated by a short straight line. Thus the fraction one-half is written $\frac{1}{2}$. The fraction one-third is written $\frac{1}{3}$. One-fourth is written $\frac{1}{4}$, and one-fifth is written $\frac{1}{5}$. In some books, such as handbooks for engineers, you may find a slanting line used between the numbers instead of a horizontal line, as, for example, 1/2, 1/3, 1/4, and 1/5.

Check Your Learning

You became acquainted with **Check Your Learning** in Part 1 of the *Practical Arithmetic* series. In this text, Part 2 of the series, you'll find more programmed statements to help you master fractions.

As in Part 1, the statements are numbered consecutively throughout the text. You'll again find the correct answer slightly below and to the right of each statement. Cover our answer with a piece of paper, and then complete the statement by writing the best answer you can in the blank provided. Now compare your answer with ours. If your answer is correct, go on to the next statement; if your answer is not correct, review the text material on which the statement is based.

1. Whenever you add, subtract, or multiply whole numbers, the result will be a __________ __________.
(whole number)

2. There are times when a whole number is not and cannot be the complete answer to a __________ problem.
(division)

3. The number that is left when a division is ended is called the __________.
(remainder)

4. The remainder in division may be expressed in the form of a ________________ and considered as part of the quotient.

(fraction)

5. When an object is divided or cut into parts, each part is (larger, smaller) than the whole.

(smaller)

6. When an object is divided into two equal parts, each part is called ____________ of the object.

(one-half)

7. If we cut an apple into three equal parts, one part would be equal to ____________ of the apple.

(one-third)

8. A fraction consists of ______________ numbers written one below the other and separated by a short straight line.

(two)

9. The fraction one-fourth is written ____________ .

($\frac{1}{4}$)

Parts of a Fraction

5. As you've already learned, every fraction must have two numbers, one above the line and the other below it. These two numbers have special names. The one that is *above* the line is called the *numerator* of the fraction, and the one *below* the line is called the *denominator*. Considered together, the numerator and denominator are called the *terms* of a fraction.

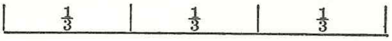
The denominator of a fraction – the number written below the line – shows into how many equal parts a thing is divided. The numerator shows how many of these equal parts are being considered. In Fig. 1*(c)*, for instance, one of the four equal parts of the circle is shaded. This part forms one-fourth of the circle and is represented by the fraction $\frac{1}{4}$. Here the denominator is 4, which denotes the number of equal parts into which the circle is divided. The numerator is 1, which denotes the single shaded part being considered. In *(d)*, two of the five equal parts are shaded; hence two-fifths of the circle is shaded and the fraction is written $\frac{2}{5}$. The denominator 5 shows that there are five equal parts; the numerator 2 shows that two of these parts – the shaded ones – are being considered.

If the unshaded portion in *(d)* is considered, the fraction is three-fifths, or $\frac{3}{5}$, as three of the five parts are not shaded. Similarly, in *(c)*, the unshaded portion is denoted by the fraction $\frac{3}{4}$. A fraction indicated by two numbers written as just shown is known as a *common fraction* to distinguish it from a decimal, or *decimal fraction,* which will be considered in detail in your next text.

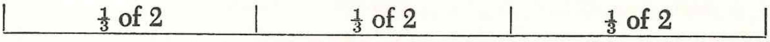
Fractions Defined as Division

6. Before you can go on with your study, it is necessary to broaden your concept of fractions. You've learned that a fraction indicates how many of a certain number of equal parts are to be considered. Thus we say that $\frac{1}{3}$ represents one of three equal parts of an object. Also, we say that when something is divided into three equal parts, it is divided by 3. Therefore, it is logical to conclude that $\frac{1}{3}$ and $1 \div 3$ are equal, because they mean the same thing. The same reasoning will convince us that $\frac{1}{2}$ means $1 \div 2$; $\frac{1}{4}$ is $1 \div 4$; and $\frac{1}{5} = 1 \div 5$. At this point you'll probably wonder about the meaning of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$.

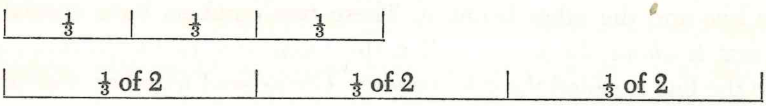
To explain what is meant by $\frac{2}{3}$, let us imagine that a line has been divided into three equal parts. Thus,



Now let's suppose that we have another line exactly twice as long as the first and divided into three equal parts. Thus,



This longer line will be called 2, since it is equal to two of the shorter lines. Each division of the longer line will be called $\frac{1}{3}$ of 2. Now let's compare the two lines, either by measuring them or by placing them one over the other, as we've done here.



Notice that one of the three equal parts of the longer line represented by $\frac{1}{3}$ of 2 is equal to two-thirds, or $\frac{2}{3}$, of the shorter line. Therefore, we can conclude that $\frac{1}{3}$ of 2, which means $2 \div 3$, equals $\frac{2}{3}$. By similar reasoning, we can establish the fact that $\frac{3}{4}$ means $3 \div 4$ and $\frac{4}{5} = 4 \div 5$.

Now we've reached a point at which it is possible to state a new and much more useful definition of a fraction. Our new definition reads like this:

A fraction is the indicated division of two numbers; the numerator is the dividend and the denominator is the divisor.

Check Your Learning

10. In a fraction, the number above the line is called the ____________________.
(numerator)

11. The number below the line is called the ____________________ of a fraction.
(denominator)

12. The numerator and the denominator of a fraction together are called the __________ of the fraction.

(terms)

13. The term of a fraction which tells the number of equal parts into which an object is divided is the ____________________ .

(denominator)

14. The term of a fraction which tells how many of the equal parts are being considered is the ____________________ .

(numerator)

15. A fraction is the indicated quotient of two numbers in which the numerator is the __________________ and the denominator is the ____________________ .

(dividend) (divisor)

The Value of a Fraction

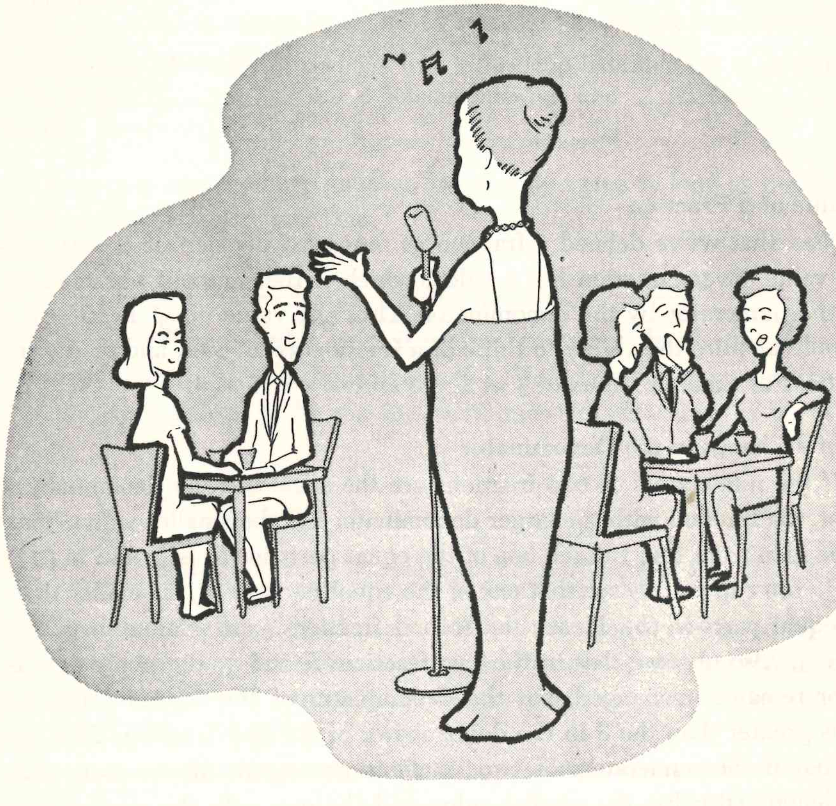
7. Now that we've defined a fraction as indicated division, it is very easy to find its value. Every fraction has a value, which is the quotient obtained by dividing the numerator by the denominator. Thus the value of $\frac{15}{3}$ is $15 \div 3 = 5$. You should acquire the ability to think of a fraction as division and of division as a fraction. For example, think of $\frac{2}{3}$ as $2 \div 3$ and of $4 \div 7$ as $\frac{4}{7}$.

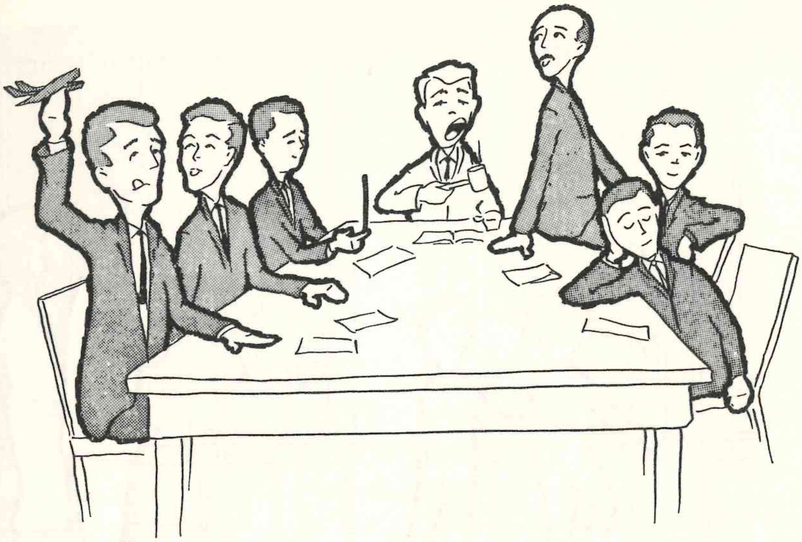
Effect of a Change in the Denominator

8. If the numerators of two fractions are the same but the denominators are different, the fraction with the larger denominator has the smaller value. This can easily be seen from Fig. 1. Each one of the equal parts in *(b)* is $\frac{1}{3}$, and in *(d)* each part is $\frac{1}{5}$. You can readily see that one of the equal parts in *(d)* is smaller than one of the equal parts in *(b)*. Hence the second fraction, $\frac{1}{5}$, is smaller than the first fraction, $\frac{1}{3}$. Also observe that in the two fractions $\frac{1}{3}$ and $\frac{1}{5}$, the numerator is the same, or remains unchanged, but the denominator of the second fraction is 5, which is greater than the 3 in the denominator of the first fraction. This goes to prove that if the numerators of two fractions are equal, the fraction with the larger denominator has the smaller value and the one with the smaller denominator has the greater value. Thus $\frac{3}{10}$ is smaller than $\frac{3}{4}$, and $\frac{3}{5}$ is larger than $\frac{3}{7}$.

Effect of a Change in the Numerator

9. If the denominators of two fractions are equal, the fraction having the larger numerator has the greater value. Consider $\frac{2}{5}$ and $\frac{4}{5}$, for instance. The first indicates two of five equal parts, such as two of the parts in Fig. 1*(d)*, whereas the second indicates four of five equal parts. It is obvious that four parts are more than two parts. Hence the fraction with the larger numerator has the larger value.





Proper Fractions and Improper Fractions

10. If the numerator of a fraction is less than the denominator, the fraction is called a *proper fraction.* Thus $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{3}{10}$, and $\frac{1}{16}$ are proper fractions. But if the numerator is equal to or greater than the denominator, the fraction is an *improper fraction.* Thus $\frac{2}{2}$, $\frac{3}{2}$, $\frac{12}{4}$, $\frac{6}{5}$, $\frac{7}{6}$, $\frac{9}{8}$, $\frac{19}{10}$, $\frac{16}{12}$, $\frac{15}{15}$, and $\frac{20}{18}$ are improper fractions.

Check Your Learning

16. The quotient obtained by dividing the numerator of a fraction by the denominator gives the ________________ of the fraction.

(value)

17. If the numerators of two fractions are equal, the fraction with the larger denominator has the smaller value and the fraction with the smaller denominator has the ________________ value.

(greater)

18. A fraction whose numerator is less than its denominator is called a ________________ fraction.

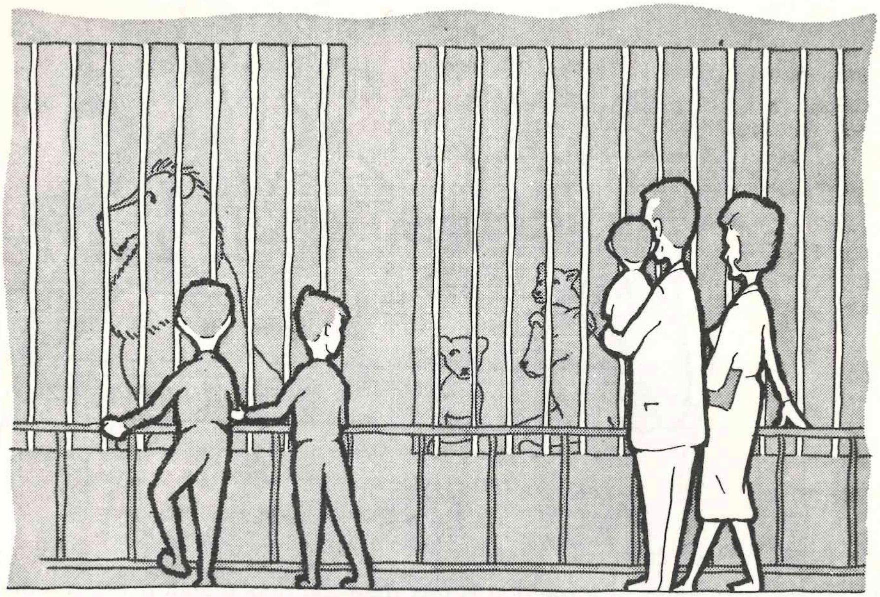
(proper)

19. If the numerator of a fraction is larger than or equal to the denominator, the fraction is called an ________________ fraction.

(improper)

Prime Numbers and Composite Numbers

11. A *prime number* is one that cannot be divided by any number except itself and 1 without a remainder. The numbers 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, and 53 are prime numbers.



Any number not a prime number is called a *composite number*, and may be considered as the product of two or more prime numbers. Thus 60 is a composite number, and is equal to the product of the prime numbers 2, 2, 3, and 5; in other words, $2 \times 2 \times 3 \times 5 = 60$.

Whole Numbers and Mixed Numbers

12. A *whole number* is a number that does not contain a fraction. For example, 2, 36, 185, and 4,063 are whole numbers. A *mixed number* is a number composed of a whole number and a fraction. For example, $3\frac{3}{8}$ is a mixed number, since it is composed of the whole number 3 and the fraction $\frac{3}{8}$. This number is read three and three-eighths. It is actually equal to $3 + \frac{3}{8}$, but the plus sign is omitted and it appears simply as $3\frac{3}{8}$. The mixed number $10\frac{5}{16}$ is read ten and five-sixteenths. As you learned earlier, a whole number is very frequently called an *integer*.

Check Your Learning

20. A whole number that cannot be divided by any number except itself and 1 without a remainder is called a ________________ number.
(prime)

21. If a whole number is not a prime number, it is a ____________________ number.
(composite)

22. A number that does not contain a fraction is called a ________________ number.
(whole)

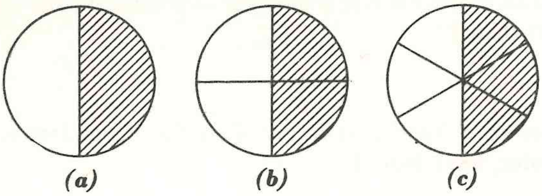


FIG. 2

23. A number composed of a whole number and a fraction is a ________________ number.

(mixed)

24. A whole number may also be referred to as an ________________ .

(integer)

Fractions Smaller or Greater Than 1

13. A fraction whose numerator and denominator are equal has a value of 1. For example, $\frac{4}{4} = 1$, $\frac{2}{2} = 1$, and $\frac{8}{8} = 1$. If the numerator is less than the denominator, the value of the fraction is less than 1. The value of each of the fractions $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{16}$, is less than 1. If the numerator of the fraction is larger than the denominator, the value of the fraction is more than 1. Thus $\frac{12}{2} = 6$, $\frac{10}{5} = 2$, and $\frac{3}{2} = 1\frac{1}{2}$.

Writing a Fraction in Different Forms

14. Because the denominator of a fraction denotes the number of equal parts into which a thing is divided, it determines the *form* of the fraction. The *form* of a fraction may be changed, however, without altering the *value* of the fraction. To clarify this, let's take a look at Fig. 2, *(a)* through *(c)*. Here you see three circles of the same size, with half of each circle shaded; that is, the shaded parts of the circles are equal. However, the circle in *(a)* is divided into two equal parts; the one in *(b)* is divided into four equal parts; and the one in *(c)* is divided into six equal parts. In *(a)*, therefore, the shaded part is one of two equal divisions, or $\frac{1}{2}$; in *(b)* the shaded part covers two of the four equal divisions, or $\frac{2}{4}$; and in *(c)* the shaded part covers three of the six equal divisions, or $\frac{3}{6}$. Since the three circles are of the same size, and since the same amount of each is shaded, it is obvious that the fractions $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ must be equal to each other, because each represents half of a circle. In other words, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$. Here we have a fraction expressed in three different forms without any change in value.

Check Your Learning

25. A fraction whose numerator and denominator are equal has a value of __________ .

(1)

26. If the numerator of a fraction is less than the denominator, the value of the fraction is (greater, less) than 1.

(less)

27. If the numerator of a fraction is larger than the denominator, the value of the fraction is (greater, less) than 1.

(greater)

28. Because the denominator of a fraction denotes the number of equal parts into which a thing is divided, it determines the _______________ of the fraction.

(form)

General Principle of Reducing Fractions

15. The principle just brought out—namely, that the form of a fraction may be changed without any change in the value of the fraction—is very important, for it leads to another principle:

If both numerator and denominator are multiplied or divided by the same number, the value of the resulting fraction is the same as that of the original fraction.

For example, suppose that both terms of the fraction $\frac{1}{2}$ are multiplied by 2. Then $\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$. The fraction $\frac{2}{4}$ has the same value as $\frac{1}{2}$, as was shown in Fig. 2. Again, take $\frac{1}{2}$, but this time multiply both terms by 3. Then $\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$, which also has the same value as $\frac{1}{2}$. Now take the fraction $\frac{3}{6}$ and divide both terms by 3. You have $\frac{3 \div 3}{6 \div 3} = \frac{1}{2}$. But it was shown that $\frac{1}{2}$ equals $\frac{3}{6}$. Thus dividing both terms by the same number does not change the value of the fraction. This process of changing the *form* of a fraction without changing its *value* is called *reduction of fractions.*

Reducing a Fraction to Higher Terms

16. You already know that the terms of a fraction are the two numbers which make up the fraction, the numerator and the denominator. If these terms are multiplied by the same number, the new terms obtained will be greater than the original terms. In other words, the fraction is changed to higher terms, which means that the numerator and the denominator are increased. *The operation of multiplying the terms of a fraction by the same number is called reducing the fraction to higher terms.* For example, $\frac{1}{6}$ can be reduced to $\frac{4}{24}$ by multiplying both terms of the fraction by 4. The operation may be written like this:

$$\frac{1}{6} = \frac{1 \times 4}{6 \times 4} = \frac{4}{24}$$

As another example, $\frac{2}{3}$ can be changed to $\frac{16}{24}$ by multiplying each term by 8. Thus

$$\frac{2}{3}=\frac{2\times 8}{3\times 8}=\frac{16}{24}$$

As a final example,

$$\frac{3}{5}=\frac{3\times 7}{5\times 7}=\frac{21}{35}$$

Reducing a Fraction to Lower Terms

17. *A fraction is reduced to lower terms by dividing both terms by the same number.* For example, $\frac{20}{24}$ is reduced to $\frac{5}{6}$ by dividing both terms by 4. Thus

$$\frac{20}{24}=\frac{20\div 4}{24\div 4}=\frac{5}{6}$$

A fraction is reduced to its lowest terms when both the numerator and the denominator cannot be divided by the same number without a remainder. For example, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{11}{24}$, and $\frac{8}{15}$ are fractions reduced to their lowest terms.

EXAMPLE 1. Reduce $\frac{12}{16}$ to its lowest terms.

SOLUTION. By trial, find the greatest number that will divide into 12 and 16 without a remainder. This number is 4.

$$\frac{12}{16}=\frac{12\div 4}{16\div 4}=\frac{3}{4}. \quad \text{Ans.}$$

EXAMPLE 2. Reduce $\frac{8}{10}$ to its lowest terms.

SOLUTION. The number that will divide into 8 and 10 without a remainder is 2.

$$\frac{8}{10}=\frac{8\div 2}{10\div 2}=\frac{4}{5}. \quad \text{Ans.}$$

Check Your Learning

29. If both the numerator and the denominator of a fraction are multiplied or divided by the same number, the resulting fraction has the same value as the __________ fraction.

(original)

30. The process of changing the form of a fraction without changing its value is called ________________ the fraction.

(reducing)

31. When the terms of a fraction are multiplied by the same number, the new terms obtained will be ________________ than the original terms.

(greater)

32. The operation of multiplying the terms of a fraction by the same number is calle reducing the fraction to ________________ terms.
(higher)

33. A fraction is reduced to lower terms by ________________ both terms of th fraction by the same number.
(dividing)

34. A fraction has been reduced to its lowest terms when both the numerator and th denominator cannot be divided by the same number without a ________________
(remainder)

Writing a Fraction with a Desired Denominator

18. In solving practical problems, it is often necessary to change a fraction t another one of equal value having a different denominator. To do this, yo *divide the desired new denominator by the denominator of the given fraction, an then multiply both terms of the given fraction by the quotient obtained.* An example or two will illustrate the method.

EXAMPLE 1. Reduce $\frac{7}{8}$ to an equal fraction having 96 for its denominator.

SOLUTION. The desired new denominator is 96 and the denominator of the give fraction is 8. Then $96 \div 8 = 12$. Now both terms of the given fraction $\frac{7}{8}$ are multiplie by the quotient 12. Thus $\frac{7 \times 12}{8 \times 12} = \frac{84}{96}$. Hence $\frac{7}{8} = \frac{84}{96}$. Ans.

EXAMPLE 2. Reduce $\frac{3}{4}$ to hundredths, that is, to a fraction that has a denominato of 100.

SOLUTION. The desired denominator is 100, and the denominator of the given fraction is 4. Then $100 \div 4 = 25$, which is the number by which both terms of the given fraction are to be multiplied; that is, $\frac{3 \times 25}{4 \times 25} = \frac{75}{100}$. Ans.

Practice Problems

Here are some fractions for you to reduce yourself. Work them out carefully. Then check your results against the answers given at the back of the text.

1. $\frac{7}{16}$ to one hundred twenty-eighths
2. $\frac{24}{132}$ to its lowest terms
3. $\frac{64}{1000}$ to its lowest terms
4. $\frac{5}{7}$ to forty-ninths
5. $\frac{13}{20}$ to thousandths
6. $\frac{3}{8}$ to sixty-fourths
7. $\frac{9}{20}$ to ten-thousandths
8. $\frac{23}{299}$ to its lowest terms
9. $\frac{17}{187}$ to its lowest terms
10. $\frac{8}{12}$ to one hundred forty-fourths

Writing an Integer as a Fraction

19. An integer, or whole number, can be reduced to a fraction having a desired denominator. You simply *multiply the whole number by the desired denomi-*

nator and write the product you obtain above the desired denominator in the form of a fraction.

EXAMPLE 1. Reduce 5 to a fraction having 4 as a denominator.

SOLUTION. Multiply 5 by 4 and place the product over the required denominator. Thus

$$5 = \frac{5 \times 4}{4} = \frac{20}{4}. \quad \text{Ans.}$$

EXAMPLE 2. Reduce 75 to a fraction having 123 as its denominator.

SOLUTION. Multiply the whole number 75 by the desired denominator 123. Then write the product you obtain above the denominator, forming the required fraction. Thus

$$75 = \frac{75 \times 123}{123} = \frac{9225}{123}. \quad \text{Ans.}$$

Reducing a Mixed Number to a Fraction

20. As you've learned, a mixed number consists of a whole number and a fraction, such as $7\frac{3}{4}$, $156\frac{15}{16}$, and $39\frac{13}{27}$. *To reduce a mixed number to an improper fraction, first multiply the whole number by the denominator of the fraction, then add the numerator of the fraction to the product you obtain, and finally write this sum above the denominator.*

EXAMPLE 1. Reduce $64\frac{7}{12}$ to an improper fraction.

SOLUTION. Follow the procedure just outlined. The product of the whole number and the denominator of the fraction is 64 × 12 = 768. To this product add the numerator of the fraction, which gives you 768 + 7 = 775. This sum written above the denominator gives $\frac{775}{12}$. Ans.

EXAMPLE 2. Reduce $8\frac{3}{4}$ to an improper fraction.

SOLUTION. Multiply the whole number 8 by the denominator 4; thus 8 × 4 = 32. To this product add the numerator, which gives you 32 + 3 = 35. Writing this sum above the denominator, you have the required improper fraction, $\frac{35}{4}$. Ans.

Check Your Learning

35. To change a fraction to another of equal value having a different denominator, you divide the desired new denominator by the denominator of the given fraction, and ____________________ both terms of the given fraction by the quotient obtained.

(multiply)

36. The fraction $\frac{2}{3}$ reduced to an equivalent fraction having 12 as its denominator becomes __________.

($\frac{8}{12}$)

37. To reduce a whole number to a fraction having a desired denominator, multiply

the whole number by the desired denominator and write the _______________ above the new denominator.

(product)

38. The whole number 6, changed to an improper fraction with a denominator of 3 is __________.

($\frac{18}{3}$)

39. A mixed number consists of the sum of a whole number and a _______________

(fraction)

Practice Problems

Reduce these mixed numbers to improper fractions.

1. $32\frac{3}{4}$
2. $18\frac{5}{6}$
3. $4\frac{5}{21}$
4. $42\frac{7}{8}$
5. $26\frac{2}{3}$
6. $5\frac{3}{10}$
7. $12\frac{13}{17}$
8. $9\frac{14}{39}$
9. $6\frac{19}{21}$
10. $17\frac{6}{11}$

Reducing an Improper Fraction

21. Frequently you'll get a result in the form of an improper fraction. Now you've already learned that integers and mixed numbers can be expressed as improper fractions. This means that every improper fraction breaks down into an integer or a mixed number. In most cases, the integer or mixed-number forms are the ones used. But sometimes the improper fraction is used instead of the mixed number.

To reduce an improper fraction to an integer or a mixed number, just divide the numerator by the denominator. Here are a few examples of the reduction o improper fractions to integers.

EXAMPLE 1. Reduce $\frac{36}{4}$ to an integer.

SOLUTION. Following the procedure just stated, divide the numerator 36 by the denominator 4, which gives you $36 \div 4 = 9$. Thus the improper fraction $\frac{36}{4}$ expressed a an integer is 9. Ans.

EXAMPLE 2. Reduce $\frac{625}{25}$ to an integer.

SOLUTION. $\begin{array}{r} 25 \\ 25\overline{)625} \\ \underline{50} \\ 125 \\ \underline{125} \end{array}$ Therefore, $\frac{625}{25} = 25$. Ans.

An improper fraction is changed to a mixed number in exactly the same way There will be a difference in the result, of course, because now the division will not be complete and there will be a remainder. For example, to reduce $\frac{35}{4}$ to a mixed number, we divide 35 by 4, obtaining a quotient of 8 and a remainder of 3. Remember that in all examples of incomplete division, including this one, the remainder does not contain the divisor even once. So you should realize that a remainder in division represents a part of the original number that has not been divided by the divisor. Earlier you learned how to express division as a fraction. Therefore, you should be able to see that the remainder divided by the divisor can be written as a fraction. This fraction is then annexed to the quotient already obtained, giving the complete quotient. From now on, in all division problems, we'll express the remainder as a part of the quotient.

Now let's get back to the example in which a quotient of 8 and a remainder of 3 were found. We write the remainder 3 divided by 4 as the fraction $\frac{3}{4}$, and add it to the 8 already in the quotient. Therefore, $\frac{35}{4} = 8 + \frac{3}{4}$, or $8\frac{3}{4}$.

Let's consider one more example.

EXAMPLE 3. Reduce $\frac{4256}{27}$ to a mixed number.

SOLUTION.

$$\begin{array}{r} 157\frac{17}{27} \\ 27\overline{)4256} \\ \underline{27} \\ 155 \\ \underline{135} \\ 206 \\ \underline{189} \\ 17 \end{array}$$

The remainder 17 divided by 27 is written as a fraction, which is added to 157 for the complete quotient, which is the required mixed number. Therefore, $\frac{4256}{27} = 157\frac{17}{27}$. Ans.

Check Your Learning

40. For every ____________ fraction there is an equivalent integer or mixed number.

(improper)

41. To reduce an improper fraction to an integer or a mixed number, just ________ the numerator by the denominator.

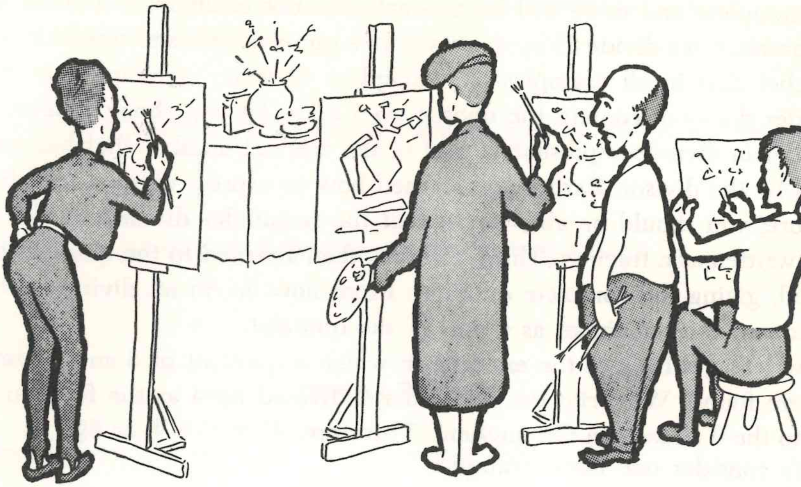
(divide)

42. The remainder of a division problem represents a part of the dividend that (does, does not) contain the divisor.

(does not)

43. The remainder in a division problem does not contain the ____________ even once.

(divisor)



Practice Problems

Reduce these improper fractions to whole or mixed numbers.

1. $\frac{135}{8}$	6. $\frac{185}{3}$
2. $\frac{214}{3}$	7. $\frac{156}{17}$
3. $\frac{740}{6}$	8. $\frac{263}{13}$
4. $\frac{275}{5}$	9. $\frac{178}{9}$
5. $\frac{256}{4}$	10. $\frac{133}{11}$

Common Denominator

22. It is possible to add and subtract fractions, but only if the fractions to b added or subtracted have the same denominator. If the fractions have differen denominators, they must be changed so that the denominators are all alike. Thu $\frac{3}{4}$ may be added to $\frac{1}{4}$ to produce $\frac{4}{4}$, but $\frac{3}{4}$ cannot be added to $\frac{7}{8}$ *as the fraction stand,* because fourths cannot be added to eighths, just as feet cannot be adde to dollars.

If several fractions have denominators that are alike, the fractions are said t have a *common denominator.* For example, the fractions $\frac{1}{11}$, $\frac{4}{11}$, $\frac{7}{11}$, and have the common denominator of 11. Fractions that have different denominator can be changed to other fractions having like denominators by reducing eac fraction to higher or lower terms. For example, $\frac{1}{2}$ and $\frac{1}{3}$ can each be reduced t sixths. Thus $\frac{1}{2} = \frac{3}{6}$, because $\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$. Also, $\frac{1}{3} = \frac{2}{6}$, because $\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$. Likewise $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{4}$ can each be reduced to twelfths. Thus $\frac{1}{2} = \frac{6}{12}$, $\frac{1}{3} = \frac{4}{12}$, and $\frac{3}{4} = \frac{9}{12}$.

Least Common Denominator

23. The common denominator of several fractions with different denominator

is any number that can be divided by each of the denominators without leaving a remainder. For example, the fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ may be changed to

$$\frac{6}{12}, \frac{4}{12}, \text{ and } \frac{3}{12}$$
$$\frac{12}{24}, \frac{8}{24}, \text{ and } \frac{6}{24}$$
$$\frac{18}{36}, \frac{12}{36}, \text{ and } \frac{9}{36}$$

or to any other form in which the denominator can be divided by 2, 3, and 4 without a remainder. However, in solving problems that involve fractions, it is generally simplest to find the smallest number which is exactly divisible by all the denominators of the given fractions. This number is called the *least common denominator*.

Suppose that the fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ are to be reduced to fractions having the same denominator. Since 2, 3, and 4 can be divided into 12 without a remainder, 12 is taken as the common denominator. Thus $12 \div 2 = 6$; then $\frac{1 \times 6}{2 \times 6} = \frac{6}{12}$. Similarly, $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$, and the fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ therefore become $\frac{6}{12}$, $\frac{8}{12}$, and $\frac{9}{12}$. The number 24 could also be used as a common denominator, because it too can be divided by 2, 3, or 4 without a remainder. The three fractions would then become $\frac{12}{24}$, $\frac{16}{24}$, and $\frac{18}{24}$, and these values would be the same as $\frac{6}{12}$, $\frac{8}{12}$, and $\frac{9}{12}$. But note that although 12 and 24 can both be used as common denominators for these three fractions, 12 is the *least* common denominator because it is the smallest number that can be divided by 2, 3, and 4 without leaving a remainder. Several fractions may have a great many different common denominators, but they can have only one least common denominator.

Finding the Least Common Denominator

24. It isn't always possible to find the least common denominator by inspection, as we just did. For this reason, a method for finding the least common denominator of any set of fractions is given here. It is important that you understand this method thoroughly. It can best be explained by applying it directly to a few examples.

EXAMPLE 1. Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

SOLUTION. The denominators of the fractions are placed in a line as in short division, and separated by commas. In this case the denominators are 4, 3, 9, and 16. The denominators are now divided by the smallest prime number other than 1 that is contained in at least two of the numbers without a remainder. First you try the number 2 because it is the smallest prime number other than 1. You see that it is contained in 4 and 16. Now you divide by 2. This gives the quotients 2 and 8, which are placed in a second line, under the numbers divided. The numbers 3 and 9, which will not contain the divisor 2 without a remainder, are brought down to the second line. Next, another prime number is found that will be contained in at least two of the numbers in the second line. The divisor 2 is again selected, because it is contained in the numbers 2 and 8. The quotients 1 and 4 are written in a third line, as well as the numbers not divisible by 2, which are again 3 and 9. The divisor 3 is chosen for the third line, since it is contained in both 3 and 9, giving the quotients 1 and 3. In the fourth line there will then appear the numbers 1, 1, 3, and 4, as shown.

$$\begin{array}{r l}
2) & \underline{4,\ 3,\ 9,\ 16} \\
2) & \underline{2,\ 3,\ 9,\ \ 8} \\
3) & \underline{1,\ 3,\ 9,\ \ 4} \\
 & 1,\ 1,\ 3,\ \ 4
\end{array}$$

The operation has been repeated until the last line contains no two numbers that can be divided by the same prime number. In this case the remaining numbers are 3 and 4, which cannot both be divided by any one prime number. Finally, the divisors and the numbers in the last line are multiplied together, their product being the least common denominator. In this example the product is $2 \times 2 \times 3 \times 1 \times 1 \times 3 \times 4 = 144$, which is the least common denominator. Ans. Ordinarily, the prime number 1 is omitted in the final result as it doesn't affect the product.

EXAMPLE 2. Find the least common denominator of $\frac{4}{9}$, $\frac{5}{12}$, and $\frac{7}{18}$.

SOLUTION. Follow the same procedure as in example 1.

$$\begin{array}{r l}
2) & \underline{9,\ 12,\ 18} \\
3) & \underline{9,\ \ 6,\ \ 9} \\
3) & \underline{3,\ \ 2,\ \ 3} \\
 & 1,\ \ 2,\ \ 1
\end{array}$$

Then the least common denominator is $2 \times 3 \times 3 \times 2 = 36$. Ans. Note that the number 1 was omitted in the final result.

Practice Problems

Find the least common denominator of the following groups of fractions.

1. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$
2. $\frac{3}{5}$, $\frac{1}{6}$, $\frac{1}{4}$
3. $\frac{5}{8}$, $\frac{2}{9}$, $\frac{1}{3}$
4. $\frac{3}{10}$, $\frac{3}{4}$, $\frac{5}{7}$

Changing Fractions to Least Common Denominators

25. You've just learned how the least common denominator of a number of fractions can be found. Did you notice that the divisors were always prime numbers? Remember, unless you use prime numbers for the divisors, you can't be certain that your result is the least common denominator. For example, when you worked example 2 in the preceding article, you might have wondered about combining the last two divisions (by 3) into a single division by 9. In other words, you might have thought, "Why not save time and effort by doing it this way?"

$$\begin{array}{r l}
2) & \underline{9,\ 12,\ 18} \\
9) & \underline{9,\ \ 6,\ \ 9} \\
 & 1,\ \ 6,\ \ 1
\end{array}$$

$$2 \times 9 \times 6 = 108$$

Of course you know that 108 is not the least common denominator and that therefore 108 is not the correct answer. Where did you go wrong? Your mistake was in using 9, which is not a prime number, as a divisor.

You've already learned how to write a fraction with a desired denominator. Now, to reduce fractions to the *least common denominator,* you will *divide the least common denominator by the denominator of each fraction in turn, and then multiply both terms, the numerator and denominator, of that fraction by the quotient.* Let's solve a few problems to see how this rule is applied.

EXAMPLE 1. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to fractions having the least common denominator.

SOLUTION. The least common denominator for the fractions is found first.

$$\begin{array}{rl} 2) & \underline{3,\ 4,\ 2} \\ & 3,\ 2,\ 1 \end{array}$$

Then $2 \times 3 \times 2 = 12$, the least common denominator. In this example, each of the three fractions must be changed to a form which will have 12 for its denominator. In the first fraction, $\frac{2}{3}$, the denominator is 3. By the rule, the least common denominator is to be divided by the denominator of the fraction, or $12 \div 3 = 4$. Both terms of the fraction are then to be multiplied by this quotient, 4; that is, $\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$.

The fraction to be considered next is $\frac{3}{4}$, whose denominator is 4. Following the same method, $12 \div 4 = 3$, and $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$.

The third fraction, $\frac{1}{2}$, has 2 for a denominator. Then $12 \div 2 = 6$, and $\frac{1 \times 6}{2 \times 6} = \frac{6}{12}$.

Hence the fractions $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ when reduced to the least common denominator become $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{6}{12}$. Ans.

EXAMPLE 2. Reduce $\frac{13}{16}$, $\frac{5}{7}$, and $\frac{11}{12}$ to the least common denominator.

SOLUTION. Again, the least common denominator is found first.

$$\begin{array}{rl} 2) & \underline{16,\ 7,\ 12} \\ 2) & \underline{\ 8,\ 7,\ \ 6} \\ & \ 4,\ 7,\ \ 3 \end{array}$$

The least common denominator is $2 \times 2 \times 4 \times 7 \times 3 = 336$.

For the fraction $\frac{13}{16}$, $336 \div 16 = 21$, and $\frac{13 \times 21}{16 \times 21} = \frac{273}{336}$.

For the fraction $\frac{5}{7}$, $336 \div 7 = 48$, and $\frac{5 \times 48}{7 \times 48} = \frac{240}{336}$.

For the fraction $\frac{11}{12}$, $336 \div 12 = 28$, and $\frac{11 \times 28}{12 \times 28} = \frac{308}{336}$.

Therefore, the three fractions reduced to the least common denominator are $\frac{273}{336}$, $\frac{240}{336}$, and $\frac{308}{336}$. Ans.

EXAMPLE 3. Reduce the fractions $\frac{4}{7}$, $\frac{3}{8}$, and $\frac{5}{9}$ to fractions having the least common denominator.

SOLUTION. If the denominators 7, 8, and 9 are set down like the denominators i examples 1 and 2, you will find that no number greater than 1 is contained in any tw of the denominators; therefore, the least common denominator is $7 \times 8 \times 9 = 504$.

For the fraction $\frac{4}{7}$, $504 \div 7 = 72$, so $\dfrac{4 \times 72}{7 \times 72} = \dfrac{288}{504}$.

For the fraction $\frac{3}{8}$, $504 \div 8 = 63$, so $\dfrac{3 \times 63}{8 \times 63} = \dfrac{189}{504}$.

For the fraction $\frac{5}{9}$, $504 \div 9 = 56$, so $\dfrac{5 \times 56}{9 \times 56} = \dfrac{280}{504}$.

Therefore, the reduced fractions are $\frac{288}{504}$, $\frac{189}{504}$, and $\frac{280}{504}$. Ans.

Practice Problems

Reduce these groups of fractions to fractions having the least common denominato

1. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$
2. $\frac{3}{16}$, $\frac{3}{4}$, $\frac{7}{32}$
3. $\frac{7}{8}$, $\frac{7}{88}$, $\frac{10}{11}$
4. $\frac{3}{5}$, $\frac{5}{8}$, $\frac{11}{40}$
5. $\frac{4}{10}$, $\frac{7}{40}$, $\frac{9}{20}$
6. $\frac{7}{15}$, $\frac{17}{30}$, $\frac{7}{10}$
7. $\frac{6}{17}$, $\frac{8}{9}$, $\frac{1}{4}$
8. $\frac{7}{10}$, $\frac{5}{6}$, $\frac{4}{9}$
9. $\frac{12}{15}$, $\frac{1}{17}$, $\frac{2}{5}$
10. $\frac{7}{9}$, $\frac{1}{18}$, $\frac{3}{27}$

Addition

Preparing Fractions for Addition

26. All the explanations which have been given so far in regard to the least common denominator and changing fractions to forms having a least common denominator are very important. These explanations lay the groundwork for adding and subtracting fractions. The reason for reducing fractions to forms having the same denominator has been explained before, but it may be well to review the matter here.

For instance, if a workman measures off the thicknesses of three pieces of material as $\frac{1}{16}$ in. (inch), $\frac{5}{16}$ in., and $\frac{7}{16}$ in., respectively, he knows that the total thickness is $\frac{13}{16}$ in. This result is obtained by adding the numerators – thus $1 + 5 + 7 = 13$ – and placing their sum over the common denominator, 16, giving $\frac{13}{16}$. But if the thicknesses measured off are, say, $\frac{1}{4}$ in., $\frac{5}{8}$ in., and $\frac{7}{16}$ in., the total thickness cannot be found by adding the numerators, because the fractions have different denominators. Before the fractions can be added, they must be changed, or reduced, to forms which have a common denominator. After they have been reduced, the process of addition is a very simple one.

Check Your Learning

44. You cannot add fractions until all of the fractions have the same______________ .

(denominator)

45. If several fractions have the same denominator, they have a____________________ denominator.

(common)

46. In the fractions $\frac{1}{17}$, $\frac{10}{17}$, and $\frac{5}{17}$, the common denominator is __________ .

(17)

47. When problems involve fractions, it is generally simplest to find the (largest, smallest) number which is exactly divisible by all the denominators of the given fractions.

(smallest)

48. The smallest number which is exactly divisible by the denominators of a group of fractions is called the ____________ __________________ denominator.

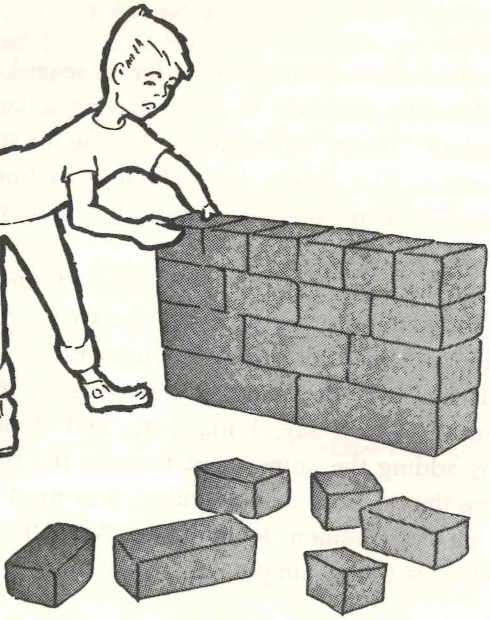
(least common)

49. The least common denominator of $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{12}$ is __________ .

(24)

Addition of Common Fractions

27. The first step in the addition of common fractions is to reduce the frac-



tions to a common denominator, using the methods we've already explained When you've done this, add the numerators of the reduced fractions and plac their sum above the common denominator. If the resulting fraction is an imprope one, reduce it to an integer or to a mixed number. Be sure to express the fractional remainder, if there is any, in its lowest terms. This series of steps is illustrated here by a few examples.

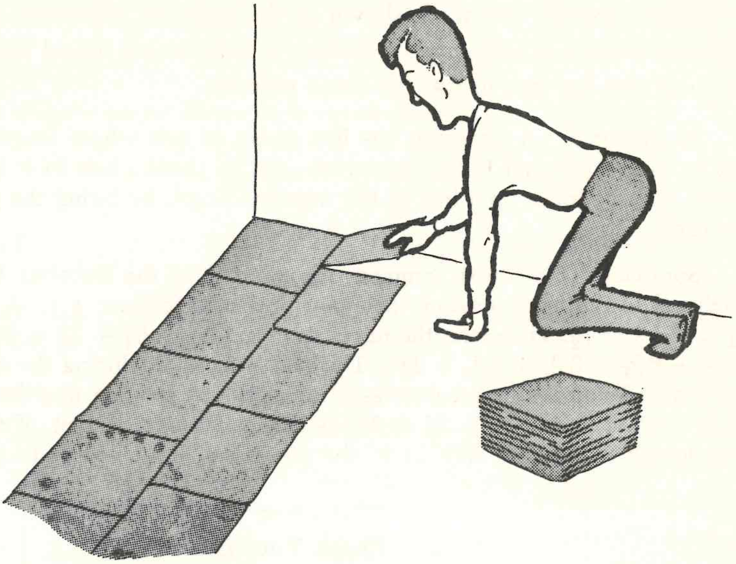
EXAMPLE 1. Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{12}$.

SOLUTION. The least common denominator of these three fractions is 12. Then $\frac{1}{2} = \frac{6}{12}$, $\frac{3}{4} = \frac{9}{12}$, and $\frac{5}{12} = \frac{5}{12}$. In their new form, the fractions have a common denominator, 12; so their numerators may be added, the sum being $6 + 9 + 5 = 20$. This sum is written above the common denominator 12, giving $\frac{20}{12}$ as the sum of the three fractions. This is an improper fraction and should be reduced to a mixed number. Thus $20 \div 12 = 1\frac{8}{12} = 1\frac{2}{3}$. Hence the sum of the three original fractions is $1\frac{2}{3}$. Ans.

EXAMPLE 2. Four crates composing a shipment had weights of $\frac{2}{3}$, $\frac{3}{5}$, $\frac{4}{7}$, and $\frac{7}{10}$ ton. What was the total weight, in tons?

SOLUTION. The total weight of the shipment is the sum of the fractions given. The least common denominator, as shown by the calculation at the right, is $5 \times 3 \times 7 \times 2 = 210$. When reduced to this common denominator, the four fractions become $\frac{2}{3} = \frac{140}{210}$; $\frac{3}{5} = \frac{126}{210}$; $\frac{4}{7} = \frac{120}{210}$; and $\frac{7}{10} = \frac{147}{210}$. The sum of the numerator of these fractions is seen to be $140 + 126 + 120 + 147 = 533$. Thus the total weight is $\frac{533}{210}$ tons. When reduced to a mixed number, this fraction becomes $2\frac{113}{210}$ tons. Ans.

$$\begin{array}{r|l} 5 & 3,\ 5,\ 7,\ 10 \\ \hline & 3,\ 1,\ 7,\ 2 \end{array}$$



Addition of Whole and Mixed Numbers

28. In the solution of practical problems, you'll frequently have to find the sum of whole numbers, mixed numbers, and fractions. In such cases, the whole numbers and the fractions are added separately, and then the two results are added.

EXAMPLE 1. What is the sum of 7, $\frac{5}{12}$, $5\frac{3}{4}$, and $8\frac{1}{2}$?

SOLUTION. The sum of the whole numbers is $7 + 5 + 8 = 20$. The sum of the fractions is $\frac{5}{12} + \frac{3}{4} + \frac{1}{2}$. Here the least common denominator is 12; so $\frac{5}{12} = \frac{5}{12}$; $\frac{3}{4} = \frac{9}{12}$; and $\frac{1}{2} = \frac{6}{12}$. Their sum is $\frac{5}{12} + \frac{9}{12} + \frac{6}{12} = \frac{20}{12} = 1\frac{8}{12} = 1\frac{2}{3}$. Then the required total is $20 + 1\frac{2}{3} = 21\frac{2}{3}$. Ans.

EXAMPLE 2. What is the total length, or overall length, of the piece of shafting shown in Fig. 3? Note that on the drawing the symbol for inches (″) is used.

SOLUTION. The total length is the sum of the three parts, measuring $12\frac{3}{4}$ in., $14\frac{5}{8}$ in., and $7\frac{5}{16}$ in. First of all, the fractions are reduced to a common denominator. Their least common denominator is 16. The whole numbers and the fractions are added separately. The sum of the fractions is $\frac{27}{16} = 1\frac{11}{16}$; the sum of the whole numbers is 33. The sum of both, which is the total, or overall, length of the shaft, is $34\frac{11}{16}$. Ans.

$$\begin{array}{rl} 12\frac{3}{4} & = 12\frac{12}{16} \\ 14\frac{5}{8} & = 14\frac{10}{16} \\ 7\frac{5}{16} & = \underline{\;7\frac{5}{16}} \\ & \;\; 33\frac{27}{16} = 34\frac{11}{16} \quad \text{Ans.} \end{array}$$

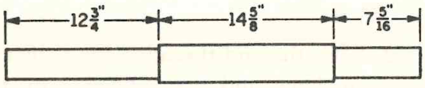


FIG. 3

Note carefully the work shown at the right. It illustrates a correct and co venient pattern to follow when adding or subtracting mixed numbers. It would b a good idea for you to use the same pattern.

EXAMPLE 3. A workman has five pieces of belt whose lengths are $9\frac{1}{2}$ ft (feet $6\frac{1}{4}$ ft, $3\frac{3}{4}$ ft, $3\frac{1}{3}$ ft, and $1\frac{5}{6}$ ft, respectively, and he needs a belt 24 ft long for some speci work. Can he make up a belt of the required length by lacing the pieces together, en to end?

SOLUTION. The least common denominator of the fractions is 12. The fraction reduced to the least common denominator are as follows: $\frac{1}{2} = \frac{6}{12}$; $\frac{1}{4} = \frac{3}{12}$; $\frac{3}{4} = \frac{9}{12}$ $\frac{1}{3} = \frac{4}{12}$; $\frac{5}{6} = \frac{10}{12}$. Therefore, the mixed numbers $9\frac{1}{2} + 6\frac{1}{4} + 3\frac{3}{4} + 3\frac{1}{3} + 1\frac{5}{6}$ are equal t $9\frac{6}{12} + 6\frac{3}{12} + 3\frac{9}{12} + 3\frac{4}{12} + 1\frac{10}{12}$. The sum is $22\frac{32}{12}$. Reducing the improper fraction $\frac{3}{1}$ to a mixed number, it becomes $2\frac{8}{12}$, in which the fraction may be reduced to $\frac{2}{3}$; thu $2\frac{8}{12} = 2\frac{2}{3}$. The sum $22 + 2\frac{2}{3} = 24\frac{2}{3}$ is the total length, in feet, of the joined pieces. A the length required is only 24 ft, the pieces can be joined to form the required belt

Ans

Check Your Learning

50. When you have found the least common denominator of the fractions to be added your next step is to add the ____________ of the fractions.
(numerators)

51. The sum of the numerators of the fractions, written above the ____________ ____________, expresses the result of the addition.
(common denominator)

52. The sum of $\frac{1}{7}$, $\frac{3}{7}$, and $\frac{2}{7}$ is ________.
($\frac{6}{7}$)

53. Whole numbers, mixed numbers, and fractions (can, cannot) be added together.
(can)

54. In the addition of whole number, mixed numbers, and fractions, the whole numbers and the fractions are added separately, and the two sums are ________.
(added)

55. The sum of 20 and $2\frac{1}{2}$ is ________.
($22\frac{1}{2}$)

Practice Problems

1. Find the sum of the following:
   (a) $\frac{4}{6}$, $\frac{7}{24}$, $\frac{5}{8}$ (b) $\frac{2}{3}$, $\frac{5}{15}$, $\frac{24}{45}$
2. The bolt shown in Fig. 4 has the end threaded for a distance of $1\frac{15}{16}$ in., is plain for $3\frac{3}{4}$ in., has a collar $\frac{17}{32}$ in. long, and has a head $\frac{7}{8}$ in. thick. What is the overall length of the bolt?

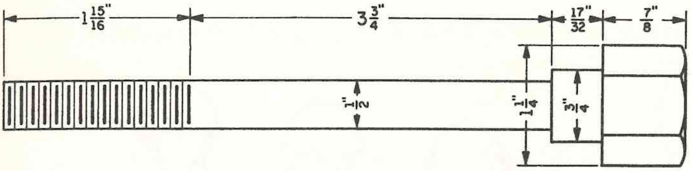


FIG. 4

3. A helper works $2\frac{3}{4}$ hr (hours) on one job, $1\frac{1}{3}$ hr on another, and $\frac{5}{12}$ hr on a third. What is the total time that he worked?

4. The weights of a number of castings are as follows: $412\frac{3}{4}$ lb (pounds), $270\frac{1}{2}$ lb, 1020 lb, $75\frac{1}{4}$ lb, and $68\frac{1}{2}$ lb. What is their total weight?

5. Find the total length of five cuts of cloth, the lengths of the cuts being as follows: $51\frac{1}{4}$ yd (yards), $51\frac{3}{8}$ yd, $55\frac{1}{4}$ yd, $53\frac{5}{8}$ yd, and $48\frac{1}{2}$ yd.

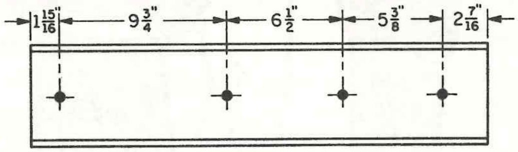


FIG. 5

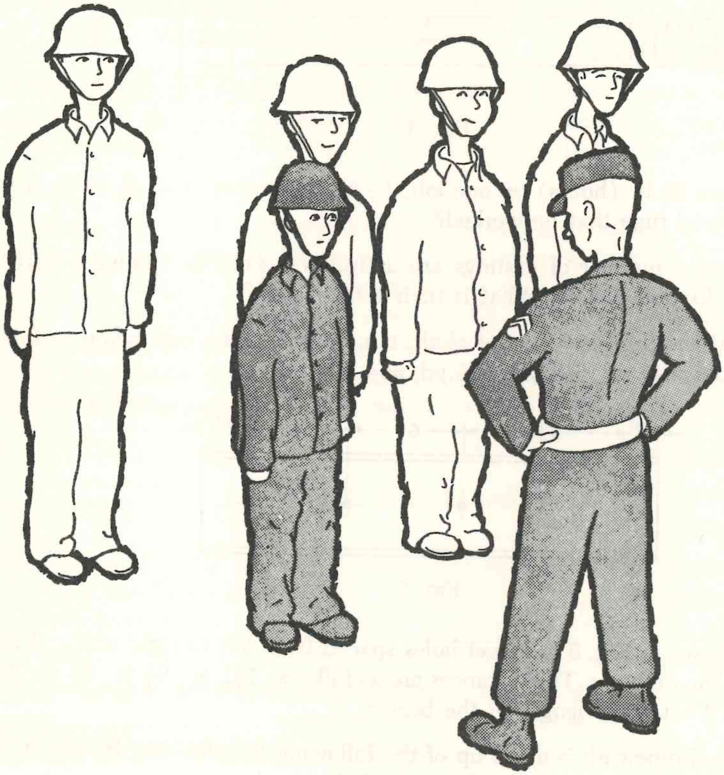
6. The beam shown in Fig. 5 has rivet holes spaced from left to right at the distances indicated in the drawing. The distances are as follows: $1\frac{15}{16}$ in., $9\frac{3}{4}$ in., $6\frac{1}{2}$ in., $5\frac{3}{8}$ in., and $2\frac{7}{16}$ in. What is the length of the beam?

7. A piece of lead pipework is made up of the following lengths: $39\frac{1}{8}$ ft, $46\frac{1}{4}$ ft, $3\frac{3}{8}$ ft, $9\frac{3}{4}$ ft, and $12\frac{1}{2}$ ft. What is the total length of the pipe?

8. During a trip, a man travels the following distances: $50\frac{5}{12}$ miles, $56\frac{3}{4}$ miles, and $85\frac{5}{6}$ miles. What is the total length of the trip?

9. Four castings of the following weights are ordered from a brass foundry: $6\frac{1}{4}$ lb, $3\frac{1}{2}$ lb, $5\frac{3}{4}$ lb, and $8\frac{7}{8}$ lb. What is the total weight of the castings?

10. The stock required for three bolts must be cut from a bar of iron. The finished lengths of the bolts are to be $2\frac{3}{16}$ in., $1\frac{13}{16}$ in., and $3\frac{1}{4}$ in. Allowing, in all, $\frac{3}{4}$ in of stock for cutting off, for forming heads, and for finishing the ends of the bolts, how long must the piece of stock be?



Subtraction

How to Prepare Fractions for Subtraction

29. Fractions, as you know, cannot be added until they are reduced to common denominator. And as you probably suspect, the same is true when comes to the subtraction of fractions. In other words, before fractions can b subtracted, the two fractions must be reduced to a common denominator.

Subtracting Common Fractions

30. After you have reduced the fractions to be subtracted to a common denominator, subtract the smaller numerator from the larger, and write the difference above the common denominator.

EXAMPLE 1. Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

SOLUTION. The least common denominator of the two fractions is 16. Then $\frac{3}{8} = \frac{6}{16}$, and $\frac{13}{16} - \frac{6}{16} = \frac{7}{16}$. Ans.

EXAMPLE 2. From a stock of aluminum sheets weighing $\frac{18}{25}$ ton, $\frac{1}{2}$ ton is removed What is the weight of the remaining sheets?

SOLUTION. The weight of sheets remaining is $\frac{18}{25} - \frac{1}{2}$. The least common denomi nator of these two fractions is 50. Then $\frac{18}{25} = \frac{36}{50}$ and $\frac{1}{2} = \frac{25}{50}$. The weight remaining is $\frac{36}{50} - \frac{25}{50} = \frac{11}{50}$. Ans.

Subtracting Mixed Numbers

31. When subtraction involves mixed numbers, you subtract the whole numbers and the fractions separately, placing the differences side by side.

EXAMPLE 1. Subtract $23\frac{4}{9}$ from $68\frac{15}{16}$.

SOLUTION. The difference of the whole numbers is $68 - 23 = 45$. The difference of the fractions is $\frac{15}{16} - \frac{4}{9}$. The least common denominator of the two fractions is 144. Then $\frac{15}{16} = \frac{135}{144}$ and $\frac{4}{9} = \frac{64}{144}$, and so $\frac{135}{144} - \frac{64}{144} = \frac{71}{144}$. When this fraction is placed next to the difference of the whole numbers, the result is $45\frac{71}{144}$. Ans.

Sometimes you'll find that the fraction in the subtrahend is larger than the one in the minuend, as for example, if $5\frac{7}{8}$ is to be subtracted from $9\frac{1}{4}$. It is obvious that $\frac{7}{8}$ cannot be taken from $\frac{1}{4}$, or $\frac{2}{8}$, because $\frac{7}{8}$ is greater than $\frac{1}{4}$. However, you'll recall that in the subtraction of whole numbers it was sometimes necessary to borrow 1 from an adjacent figure to make the subtraction possible. This can be done with fractions too. You borrow 1 from the 9 and add it to the fraction $\frac{1}{4}$. Now $1 = \frac{4}{4}$ and $\frac{4}{4} + \frac{1}{4} = \frac{5}{4}$. Since 1 was taken from 9, only 8 remains. In other words, we have changed $9\frac{1}{4}$ to an equal value, $8\frac{5}{4}$. Thus the subtraction now becomes $8\frac{5}{4} - 5\frac{7}{8}$. The common denominator of the fractions is 8, and $\frac{5}{4} = \frac{10}{8}$. Then $\frac{10}{8} - \frac{7}{8} = \frac{3}{8}$. The difference between 8 and 5 is 3. Therefore, the answer is $9\frac{1}{4} - 5\frac{7}{8} = 8\frac{5}{4} - 5\frac{7}{8} = 8\frac{10}{8} - 5\frac{7}{8} = 3\frac{3}{8}$.

EXAMPLE 2. What is the difference between $12\frac{3}{5}$ and $6\frac{7}{9}$?

SOLUTION. Since $\frac{7}{9}$ is greater than $\frac{3}{5}$, it cannot be subtracted from $\frac{3}{5}$. However, 1 can be borrowed from 12 and added to $\frac{3}{5}$. Because 1 has been taken from 12, the remainder is 11. So instead of $12\frac{3}{5}$, we have the equal value $11\frac{8}{5}$. The subtraction then becomes $11\frac{8}{5} - 6\frac{7}{9}$. The common denominator of the fractions is 45; then $\frac{8}{5} = \frac{72}{45}$ and $\frac{7}{9} = \frac{35}{45}$. The difference is $\frac{72}{45} - \frac{35}{45} = \frac{37}{45}$, which is placed next to the difference of the whole numbers, or $11 - 6 = 5$. Hence $12\frac{3}{5} - 6\frac{7}{9} = 5\frac{37}{45}$. Ans.

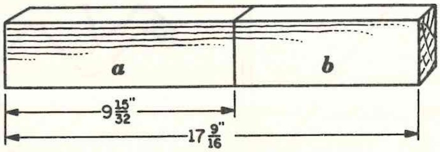


FIG. 6

EXAMPLE 3. A block of wood $17\frac{9}{16}$ in. long, shown in Fig. 6, has a piece $9\frac{15}{32}$ in. long sawed off. What is the length of the remaining piece *b*, disregarding the thickness of the saw cut?

SOLUTION. The common denominator of the fractions is 32.

minuend	$17\frac{18}{32}$
subtrahend	$9\frac{15}{32}$
difference	$8\frac{3}{32}$

That is, the remaining portion b is $8\frac{3}{32}$ in. long. Ans.

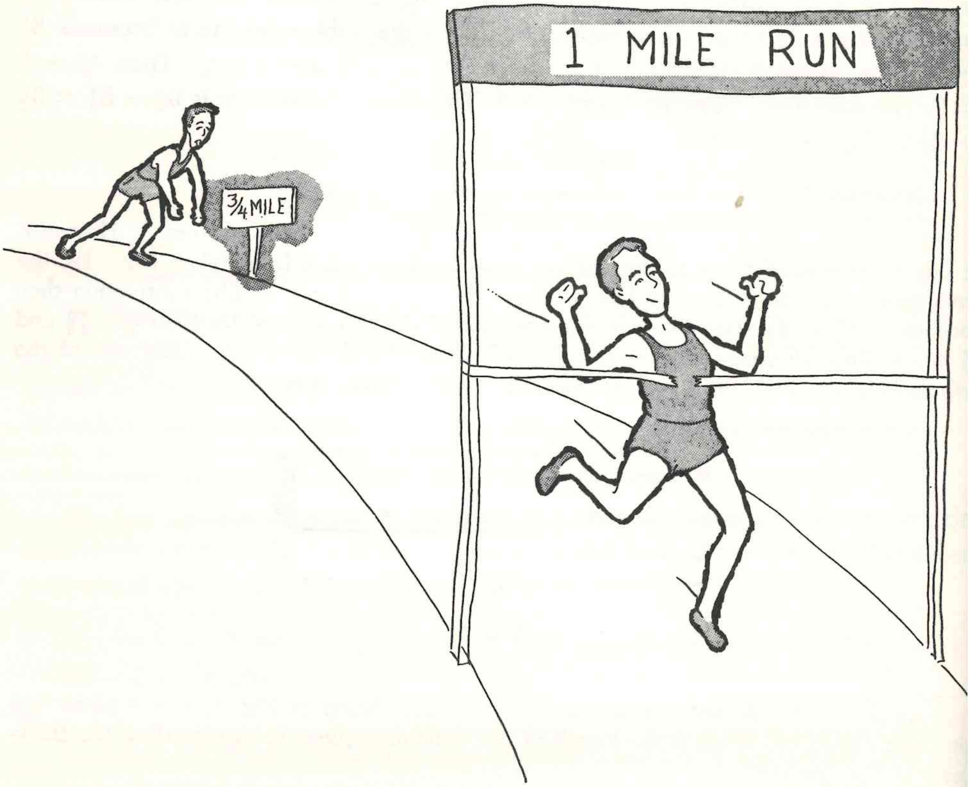
EXAMPLE 4. A piece $4\frac{7}{16}$ in. long is cut from a bar $9\frac{1}{4}$ in. long. What is the length of the remaining piece?

SOLUTION. The common denominator of the fractions is 16. Because the fraction in the subtrahend is greater than the fraction in the minuend, it cannot be subtracted as it stands. Therefore, 1, or $\frac{16}{16}$, is taken from the 9 in the minuend and added to the $\frac{4}{16}$; thus $\frac{4}{16} + \frac{16}{16} = \frac{20}{16}$. Subtracting $\frac{7}{16}$ from $\frac{20}{16}$ leaves a remainder of $\frac{13}{16}$. Since 1 was taken from 9, 8 remains. 4 from 8 = 4; $4 + \frac{13}{16} = 4\frac{13}{16}$ in., the length remaining. Ans.

minuend	$9\frac{4}{16}$	or	$8\frac{20}{16}$
subtrahend	$4\frac{7}{16}$		$4\frac{7}{16}$
difference	$4\frac{13}{16}$		$4\frac{13}{16}$

Subtracting a Fraction from an Integer

32. When a fraction or a mixed number is to be subtracted from a whole number, there will be no fraction in the minuend. And so we must borrow 1 from the whole number in the minuend and change it to a fraction having the



same denominator as the fraction in the subtrahend. Then the subtraction is performed in the usual way.

EXAMPLE 1. From 7 take $\frac{5}{8}$.

SOLUTION. There is no fraction in the minuend 7, and so it is necessary to take 1 from 7 to form the fraction whose denominator is the same as that of the fraction to be subtracted, or 8. Instead of 7 it is possible to write $6 + 1$, or $6 + \frac{8}{8}$, or simply $6\frac{8}{8}$.

$$\begin{array}{rr} \text{minuend} & 7 = 6\frac{8}{8} \\ \text{subtrahend} & \underline{\frac{5}{8}} \\ \text{difference} & 6\frac{3}{8} \end{array}$$

That is, $7 - \frac{5}{8} = 6\frac{3}{8}$. Ans.

EXAMPLE 2. From 9 take $8\frac{3}{16}$.

SOLUTION. There is no fraction in the minuend from which to subtract the fraction in the subtrahend, and so 1, or $\frac{16}{16}$, is taken from 9. Thus $9 = 8\frac{16}{16}$, and $\frac{16}{16} - \frac{3}{16} = \frac{13}{16}$. Since 1 was taken from 9, only 8 is left, and 8 from $8 = 0$. So $9 - 8\frac{3}{16} = \frac{13}{16}$. Ans.

$$\begin{array}{rrcr} \text{minuend} & 9 & \text{or} & 8\frac{16}{16} \\ \text{subtrahend} & \underline{8\frac{3}{16}} & & \underline{8\frac{3}{16}} \\ \text{difference} & \frac{13}{16} & & \frac{13}{16} \end{array}$$

Check Your Learning

56. Fractions to be subtracted (must, must not) have a common denominator.

(must)

57. Once the fractions have been reduced to a common denominator, subtract the smaller ____________________ from the larger.

(numerator)

58. After you have subtracted the smaller numerator from the larger, write the ____________________ above the common denominator.

(difference)

59. If $\frac{2}{7}$ is subtracted from $\frac{6}{7}$, the result is __________.

($\frac{4}{7}$)

60. When mixed numbers are subtracted, the whole numbers and the fractions are subtracted separately, and the ____________________ are placed side by side.

(differences)

61. The difference between $6\frac{3}{8}$ and $2\frac{2}{8}$ is __________.

($4\frac{1}{8}$)

62. In the subtraction of mixed numbers where the fraction in the subtrahend is larger than the fraction in the minuend, it is necessary to ____________________ from the whole number.

(borrow)

63. Before a fraction can be subtracted from a whole number, the whole number mus be changed to a ________________ number.

(mixed)

64. Changing the whole number 9 to a mixed number in which the denominator of th fraction is 12 results in the mixed number ____________.

($8\frac{12}{12}$)

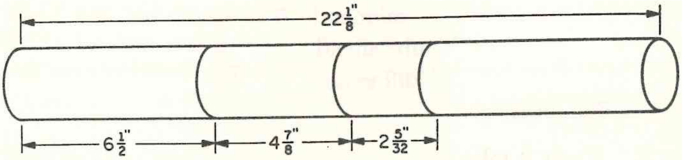


FIG. 7

Practice Problems

1. In the following examples, subtract *a)* $\frac{4}{30}$ from $\frac{5}{10}$; *b)* $13\frac{1}{4}$ from $30\frac{1}{2}$; *c)* $12\frac{1}{8}$ from 27

2. A bar of iron $22\frac{1}{8}$ in. long, as shown in Fig. 7, has three pieces cut from it, measu ing $6\frac{1}{2}$ in., $4\frac{7}{8}$ in., and $2\frac{5}{32}$ in. in length. What length of the original bar remains (Add the lengths of the three pieces that are cut off and subtract their sum fro the total length.)

3. A piece of cast iron $4\frac{1}{8}$ in. thick was planed down to a thickness of $3\frac{31}{32}$ in. Wha thickness of metal was removed?

4. At the beginning of the week an engineer had on hand $48\frac{5}{6}$ gal (gallons) of cylin der oil. During the week the following quantities were used: $2\frac{1}{4}$ gal during the fir three days, $\frac{3}{4}$ gal on the fourth day, 1 gal on the fifth day, $\frac{1}{2}$ gal on the sixth day and not any on the seventh day. How much oil remained at the end of the week

5. The main line shaft in a manufacturing plant is driven by an engine capable developing $250\frac{2}{3}$ hp (horsepower). To operate all the machinery requires $210\frac{3}{4}$ hp and to overcome the friction of the shafting and bearings requires $25\frac{1}{8}$ hp. Wha surplus power is the engine capable of developing in case it is required?

6. On a certain day a power plant had on hand $284\frac{1}{3}$ tons of coal. During the day 25 tons of coal were received, and $50\frac{1}{6}$ tons were used. How much coal was left at th end of the day?

7. A filling box in a textile mill contains $10\frac{3}{4}$ lb of yarn. When $7\frac{13}{16}$ lb of yarn is re moved from the box, how many pounds remain?

8. At the beginning of a voyage of a large steamship, an engineer's storekeeper ha on hand $50\frac{7}{12}$ lb of various sizes of rod packing. During a period of 6 days, all th hoisting engines, anchor windlasses, steering engines, and other auxiliary machiner were repacked, the quantity of rod packing used each day being as follows:

Monday	$6\frac{1}{8}$ lb	Thursday	$6\frac{5}{8}$ lb
Tuesday	$5\frac{3}{4}$ lb	Friday	$5\frac{7}{8}$ lb
Wednesday	$6\frac{1}{4}$ lb	Saturday	$4\frac{1}{2}$ lb

How many pounds of packing remained on hand?

9. A steamship has to run a distance of $498\frac{3}{4}$ miles. If during the first day she made $374\frac{2}{5}$ miles, how many more miles must she make to complete the voyage?

10. A certain factory requires a total motor installation of $27\frac{1}{2}$ hp. Three motors have been purchased, one rated at 5 hp, a second at $\frac{1}{4}$ hp, and a third at $\frac{2}{3}$ hp. What must be the combined horsepower of the motors that still remain to be purchased, to give the required total of $27\frac{1}{2}$ hp?

Multiplication

Getting Ready to Multiply Fractions

33. Fractions may be multiplied by whole numbers, by mixed numbers, or by other fractions, and the result of the multiplication is called the *product*, just as in the multiplication of whole numbers. The times sign is used to indicate a multiplication involving a fraction just as it was used for whole numbers. For example, $\frac{2}{3} \times \frac{3}{4}$ denotes the product of $\frac{2}{3}$ and $\frac{3}{4}$. Sometimes the word *of* is used instead of the multiplication sign. Thus $\frac{2}{3}$ of $\frac{3}{4}$ means exactly the same as $\frac{2}{3} \times \frac{3}{4}$ or $\frac{2}{3}$ times $\frac{3}{4}$. The multiplications $\frac{5}{6} \times 8$, $2\frac{2}{3} \times 9$, and $5\frac{3}{8} \times 8\frac{1}{2}$ are examples of multiplications that involve fractions, whole numbers, and mixed numbers. If the product of any multiplication contains a fraction, this fraction should be reduced to its lowest terms. Thus if the result contains the fraction $\frac{12}{18}$, it should be reduced to $\frac{2}{3}$.

Multiplying Common Fractions

34. Multiplying one fraction by another is a simpler operation than adding or subtracting fractions, because in multiplication of fractions, it is not necessary to reduce the fractions to a common denominator. *To multiply either proper or improper fractions, you merely multiply the numerators together to form a new numerator, and then multiply the denominators together to form a new denominator.* For example, suppose that the product of $\frac{3}{4}$ and $\frac{5}{8}$ is to be found. The product of the two numerators is $3 \times 5 = 15$, and the product of the two denominators is $4 \times 8 = 32$. Therefore, $\frac{3}{4} \times \frac{5}{8} = \frac{15}{32}$. If either or both of the fractions to be multiplied are improper, the same rule is followed. Thus,

$$\frac{12}{7} \times \frac{4}{5} = \frac{12 \times 4}{7 \times 5} = \frac{48}{35} \qquad \text{and} \qquad \frac{8}{5} \times \frac{11}{7} = \frac{8 \times 11}{5 \times 7} = \frac{88}{35}$$

EXAMPLE 1. What is the product of $\frac{8}{9}$ and $\frac{5}{7}$?

SOLUTION. The product of the numerators is $8 \times 5 = 40$, and that of the denominators is $9 \times 7 = 63$. Hence the product of the two fractions is $\frac{40}{63}$. Ans.

EXAMPLE 2. Find the result of $\frac{20}{11} \times \frac{4}{3}$.

SOLUTION. The product of the numerators is $20 \times 4 = 80$, and that of the denominators is $11 \times 3 = 33$. So the result is the fraction $\frac{80}{33}$, which reduces to $2\frac{14}{33}$. Ans.

Multiplying a Fraction and an Integer

35. *If the product of a fraction and a whole number is required, multiply the whole number by the numerator of the fraction and write the product over the denominator of the fraction.*

EXAMPLE. Find the product of $8 \times \frac{3}{4}$.

SOLUTION. The product of the whole number and the numerator is $8 \times 3 = 24$, and 24 is written above the denominator, so that $\frac{24}{4}$ is the result. This fraction reduces to 6, because $24 \div 4 = 6$. Hence $8 \times \frac{3}{4} = 6$. Ans.

Notice that the same result would be obtained if you considered 8 as the fraction $\frac{8}{1}$ and followed the procedure outlined in the preceding article.

Multiplying a Fraction and a Mixed Number

36. *If a fraction is to be multiplied by a mixed number,* the simplest method is to *reduce the mixed number to an improper fraction and then find the product of the two fractions.*

EXAMPLE. Find the product of $\frac{7}{8}$ and $18\frac{2}{3}$.

SOLUTION. First reduce the mixed number to an improper fraction. Thus $18\frac{2}{3} = \frac{3 \times 18 + 2}{3} = \frac{54 + 2}{3} = \frac{56}{3}$. Then $\frac{7}{8} \times \frac{56}{3} = \frac{392}{24} = \frac{49}{3} = 16\frac{1}{3}$. Ans.

Multiplying a Mixed Number by a Whole Number

37. *If a mixed number is to be multiplied by an integer,* you can, *reduce the mixed number to an improper fraction and then multiply this fraction by the whole number,* as explained previously.

EXAMPLE 1. What is the product of $12\frac{1}{8}$ and 27?

SOLUTION. The mixed number reduced to an improper fraction becomes $\frac{12 \times 8 + 1}{8} = \frac{97}{8}$. Then $27 \times \frac{97}{8} = \frac{27 \times 97}{8} = \frac{2619}{8} = 327\frac{3}{8}$. Ans.

Another way of solving such a problem is to *multiply the whole number part of the mixed number by the whole number multiplier and then the fractional part of the mixed number by the whole number multiplier, and then to add the two products.*

EXAMPLE 2. Find the product of $12\frac{1}{8}$ and 27 by the method just described.

SOLUTION. First multiply 12 by 27. The product is 324. Next, multiply $\frac{1}{8}$ by 27. This product is $\frac{27}{8}$, or $3\frac{3}{8}$. Then add the two products, 324 and $3\frac{3}{8}$. The result is $327\frac{3}{8}$, just as was found by the first method. Ans.

Multiplying Several Numbers and Fractions

38. In the examples we've used to show you how to multiply fractions by whole numbers or by mixed numbers, only two numbers have been used, to simplify the explanations. But these same methods are used in multiplications involving several expressions. For example, let's consider the problem of finding the product of $\frac{3}{4}$, $5\frac{1}{2}$, and 12. Here we have a multiplication involving a fraction, a mixed number, and a whole number. We find the product of the fraction and the mixed number and then multiply that product by the whole number. Thus $\frac{3}{4} \times 5\frac{1}{2} = \frac{3}{4} \times \frac{11}{2} = \frac{33}{8}$. Then $\frac{33}{8} \times 12 = \frac{33 \times 12}{8} = \frac{396}{8} = 49\frac{1}{2}$. It makes no difference, however, in what order the multiplications are performed. The whole number and the mixed number may be multiplied, and their product then multiplied by the fraction; or the fraction and the whole number may be multiplied, and their product then multiplied by the mixed number.

EXAMPLE. Find the result of $\frac{2}{5} \times \frac{17}{4} \times 9 \times 3\frac{1}{2}$.

SOLUTION. First, $\frac{2}{5} \times \frac{17}{4} = \frac{2 \times 17}{5 \times 4} = \frac{34}{20} = \frac{17}{10}$. Next, $\frac{17}{10} \times 9 = \frac{17 \times 9}{10} = \frac{153}{10}$. Finally, $\frac{153}{10} \times 3\frac{1}{2} = \frac{153}{10} \times \frac{7}{2} = \frac{153 \times 7}{10 \times 2} = \frac{1071}{20} = 53\frac{11}{20}$. Ans.

Check Your Learning

65. To multiply fractions, it (is, is not) necessary that the fractions have a common denominator.

(is not)

66. The result obtained when two fractions are multiplied is called the ____________.

(product)

67. To find the product of fractions, first multiply the ____________________ and then multiply the ____________________.

(numerators) (denominators)

68. The product of $\frac{2}{3}$ and $\frac{3}{5}$ is __________.

($\frac{6}{15}$ or $\frac{2}{5}$)

69. To multiply a fraction and a mixed number, reduce the mixed number to a(n) ________________ ________________.

(improper fraction)

70. To find the product of a fraction and a whole number, multiply the whole number by the ____________________ of the fraction.

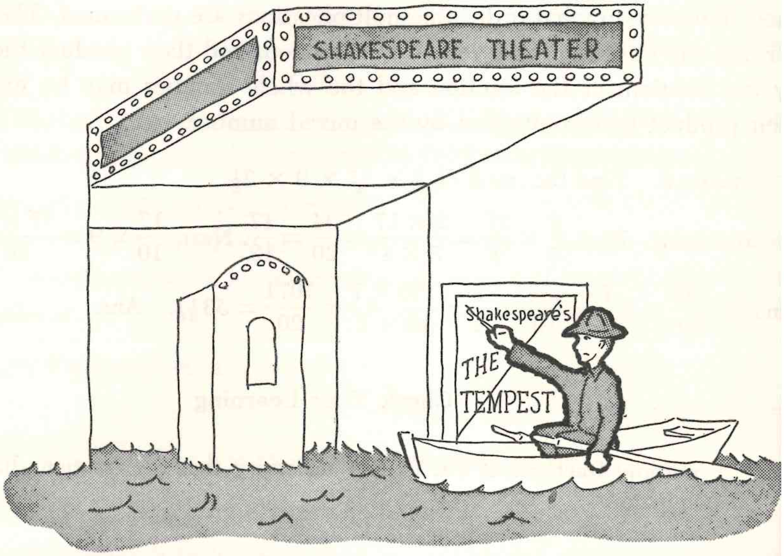
(numerator)

71. In the multiplication of a whole number and a fraction, the product of the wh number and the numerator of the fraction forms the ______________ the final product.

(numerator)

72. The denominator of the product of a whole number and a fraction is the same as t ______________ of the original fraction.

(denominator)



Cancellation

39. We've already shown that, when several fractions are multiplied t gether, the product of the numerators is written over the product of the denom nators to form the numerator and denominator of the result. It follows, there fore, that $\frac{3}{4} \times \frac{8}{9} \times \frac{10}{16}$ may be written $\frac{3 \times 8 \times 10}{4 \times 9 \times 16}$. That is, if a number of fractio are to be multiplied together, they may be represented as a single fraction i which all the numerators, connected by multiplication signs, are written abo the line, and all the denominators, also connected by multiplication signs, a written below it.

When the fractions are arranged in this way, a "shortcut" may be used to fin the product; this shortcut is called *cancellation.* Instead of multiplying the nume ators and denominators separately, it may be possible to *divide the numerato and the denominators by the same number,* just as we do in reducing a fractio to its lowest terms. This changes the form of the fraction, but not its value.

In the expression $\frac{3 \times 8 \times 10}{4 \times 9 \times 16}$, note that there is a 3 in the numerator and a 9 in the denominator, and that it is possible to divide both the 3 and the 9 by 3. To show that this division has been completed, and to prevent error through use of the same figures again in later divisions, we draw a slanting line through the figures 3 and 9. This line indicates that neither figure may be used again in another division. The quotient of each division is written above the number in the numerator and below the number in the denominator. Using the expression at the beginning of this paragraph, the first step in the cancellation process is

$$\frac{\overset{1}{\cancel{3}} \times 8 \times 10}{4 \times \underset{3}{\cancel{9}} \times 16}$$

showing that the divisor 3 is contained 1 time in 3 and 3 times in 9.

Observe that there is an 8 above the line and a 16 below it. Both of these numbers can be divided by 8. So the next step is

$$\frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{8}} \times 10}{4 \times \underset{3}{\cancel{9}} \times \underset{2}{\cancel{16}}}$$

Now, the 10 above the line and the 4 below it are both divisible by 2. Hence the next step in the cancellation is

$$\frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{8}} \times \overset{5}{\cancel{10}}}{\underset{2}{\cancel{4}} \times \underset{3}{\cancel{9}} \times \underset{2}{\cancel{16}}}$$

Further cancellations are not possible, because there are no numbers above and below the line that can be divided by the same number without leaving a remainder.

Now all the uncrossed, or uncanceled, numbers in the numerator are multiplied together, or $1 \times 1 \times 5 = 5$. Next, all the uncrossed numbers in the denominator are multiplied together, or $2 \times 3 \times 2 = 12$. The first product is written above the second, giving $\frac{5}{12}$ as the result. We can easily prove that this answer is correct. For example, $\frac{3 \times 8 \times 10}{4 \times 9 \times 16} = \frac{240}{576}$, which reduces to $\frac{5}{12}$. This shows that dividing the numbers above and below the line will give the same result as taking the product of all the numerators and dividing it by the product of all the denominators. It demonstrates also how cancellation simplifies the solution.

When the slanting line is drawn through a number, the number is said to be *canceled.* In cancellation, if the quotient is 1, it is generally not written at all, but is understood; thus the canceled expression in our example would actually be written

$$\frac{\cancel{3}\times\cancel{8}\times\overset{5}{\cancel{10}}}{\underset{2}{\cancel{4}}\times\underset{3}{\cancel{9}}\times\underset{2}{\cancel{16}}}=\frac{5}{12}.$$

Observe how these principles are applied in the following examples:

EXAMPLE 1. What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION. $\frac{4}{16}\times\frac{7}{8}=\frac{4\times 7}{16\times 8}=\frac{28}{128}=\frac{7}{32}.$ Ans.

Or, by cancellation,

$$\frac{\cancel{4}\times 7}{\underset{4}{\cancel{16}}\times 8}=\frac{7}{4\times 8}=\frac{7}{32}.\quad\text{Ans.}$$

EXAMPLE 2. What is $\frac{4}{8}$ of $\frac{3}{4}$ of $\frac{16}{32}$?

SOLUTION. $\frac{\cancel{4}\times 3\times\cancel{16}}{8\times\cancel{4}\times\underset{2}{\cancel{32}}}=\frac{3}{8\times 2}=\frac{3}{16}.$ Ans.

EXAMPLE 3. What is the product of $32\frac{5}{8}$ and 24?

SOLUTION. The mixed number $32\frac{5}{8}$ when reduced to an improper fraction becomes $\frac{32\times 8+5}{8}=\frac{261}{8}$. Multiplying the improper fraction by the whole number,

$$\frac{261}{8}\times 24=\frac{261\times 24}{8}=\frac{261\times\overset{3}{\cancel{24}}}{\cancel{8}}=261\times 3=783.\quad\text{Ans.}$$

The solution of this example may also be found by separate multiplication and addition of the two products. Thus,

$$32\times 24=768,\text{ and }\frac{5}{8}\times 24=\frac{5\times\overset{3}{\cancel{24}}}{\cancel{8}}=15$$

Now add these products, obtaining 768 + 15 = 783. That is,

$$32\tfrac{5}{8}\times 24=783.\quad\text{Ans.}$$

Some Cautions About Cancellation

40. You must remember that cancellation can be used only when all of the numbers in both the numerator and the denominator of an expression are *multiplied together*. If addition or subtraction is indicated in either term of the fraction, cancellation *cannot* be used. Thus it would be wrong to use cancellation in the expression $\frac{3\times 10-4+6}{8-4}$. Thus $\frac{3\times 10-4+6}{8-4}=\frac{30-4+6}{4}=\frac{26+6}{4}=\frac{32}{4}=8.$

Also, where cancellation can be used, you should cancel as far as possible. Frequently, some of the numbers that were written in place of canceled numbers can themselves be canceled into other numbers. For example, in the expression

$$\frac{\cancel{4}\times\overset{6}{\cancel{18}}\times 5}{\cancel{3}\times\underset{5}{\cancel{20}}\times 12}$$

we've canceled 4 into 20 and 3 into 18. We can continue the process by canceling 6 into 12 and 5 into 5. Thus the complete work would be shown like this:

$$\frac{\cancel{4}\times\overset{\cancel{6}}{\cancel{18}}\times\cancel{5}}{\cancel{3}\times\underset{\cancel{5}}{\cancel{20}}\times\underset{2}{\cancel{12}}}=\frac{1}{2}$$

You can tell that we've canceled as much as possible because the result is expressed in its lowest terms. If the final cancellation had not been performed, the result would have been $\frac{30}{60}$, which equals $\frac{1}{2}$ when reduced to lowest terms.

Finally, all results should be given in lowest terms. Thus $\frac{30}{60}$, $\frac{15}{30}$, $\frac{5}{10}$, or $\frac{3}{6}$ are not satisfactory answers, because they are not expressed in lowest terms.

Remember, then, an answer in fractional form obtained after performing *any operation* should be expressed *in lowest terms.*

Check Your Learning

73. In cancellation, it is necessary to ____________________ the numerator and the denominator by the same number.

(divide)

74. Numbers may be canceled if they can be divided by the same number without leaving a ____________________.

(remainder)

75. Cancellation may be used only when all the numbers in the numerator and the denominator are ____________________ together.

(multiplied)

Practice Problems

1. Find the product of each of these multiplications:

a) $9 \times \frac{4}{13}$

b) $8 \times \frac{5}{16}$

c) $1\frac{3}{8} \times \frac{4}{18} \times \frac{8}{3}$

d) $2\frac{5}{7} \times \frac{21}{32} \times \frac{8}{45}$

e) $\frac{3}{4} \times \frac{7}{10} \times \frac{5}{14}$

f) $\frac{2}{3} \times \frac{9}{16} \times \frac{24}{18}$

2. How long must a piece of stock be to make 4 bolts, if each bolt requires a piece of stock $3\frac{7}{8}$ in. long, including the allowance for cutting?

3. Eleven holes are spaced equally in a straight line, as in Fig. 8. What is the distance between the centers of the end holes *a* and *b*, if the spacing from center to center of two adjacent holes is $3\frac{5}{16}$ in.? (Note that the number of spaces is one less than the number of holes.)

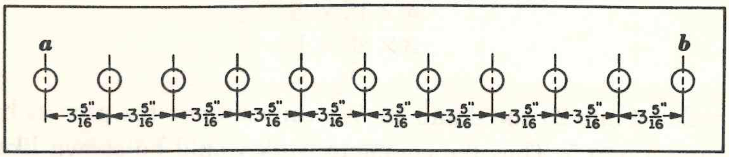


FIG. 8

4. A single belt can transmit $107\frac{2}{3}$ hp, but as more power is needed, a double belt of t same width is substituted for it. Suppose that the double belt is capable of trai mitting $1\frac{3}{7}$ times as much power as the single belt; how many horsepower can used after the change?
5. At $303\frac{3}{5}$ lb per mile, what is the weight of $2\frac{5}{8}$ miles of copper wire?
6. The grate of a steam boiler contains $20\frac{1}{2}$ sq ft (square feet). If the boiler bur $8\frac{3}{10}$ lb of coal an hour per square foot of grate area and can evaporate $7\frac{1}{2}$ lb of wat an hour per pound of coal burned, how many pounds of water are evaporated by t boiler in 1 hr?
7. Water is discharged by a return pipe at the rate of $73\frac{2}{5}$ gal per hour. If the d charge takes place during a period of $10\frac{1}{5}$ hr per day, how many gallons will be d charged in $9\frac{3}{4}$ days?
8. A foundry turns out 85 cast-iron pulleys weighing $62\frac{5}{8}$ lb apiece. What is the tot amount of metal used?

Division

The Simplicity of Division

41. Fractions may be divided by or into whole numbers, mixed numbers, fractions. Division that involves fractions is quite simple. All you do is invert t divisor and then proceed as in multiplication. The various types of multiplicatio that you will meet have already been thoroughly explained. All the instructio given for multiplication of fractions must be applied here. The procedure f division of fractions can be stated very briefly: *Invert the divisor and multipl*

A few examples should make the method clear.

EXAMPLE 1. Divide $\frac{3}{4}$ by $\frac{5}{16}$.

SOLUTION. Here the divisor is $\frac{5}{16}$, which, when it is inverted, becomes $\frac{16}{5}$. Then is multiplied by $\frac{16}{5}$. Thus $\frac{3}{4} \div \frac{5}{16} = \frac{3}{\cancel{4}} \times \frac{\overset{4}{\cancel{16}}}{5} = \frac{12}{5} = 2\frac{2}{5}$. Ans.

EXAMPLE 2. What is the value of $3 \div \frac{3}{4}$?

SOLUTION. Inverting the divisor $\frac{3}{4}$ gives $\frac{4}{3}$. So $3 \div \frac{3}{4} = 3 \times \frac{4}{3} = 4$. Ans.

EXAMPLE 3. Find the value of $\frac{3}{4} \div 3$.

SOLUTION. Here the divisor is 3, or $\frac{3}{1}$, which when inverted becomes $\frac{1}{3}$. Then $\frac{3}{4} \div 3 = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$. Ans.

EXAMPLE 4. What is the result of $13\frac{3}{4} \div 2\frac{1}{2}$?

SOLUTION. Mixed numbers must be reduced to improper fractions. So we have $13\frac{3}{4} = \frac{55}{4}$ and $2\frac{1}{2} = \frac{5}{2}$. The divisor is $\frac{5}{2}$. Inverting it gives $\frac{2}{5}$. Then

$$13\frac{3}{4} \div 2\frac{1}{2} = \frac{55}{4} \div \frac{5}{2} = \frac{\overset{11}{\cancel{55}}}{\underset{2}{\cancel{4}}} \times \frac{\cancel{2}}{\cancel{5}} = \frac{11}{2} = 5\frac{1}{2}. \quad \text{Ans.}$$

EXAMPLE 5. A bar of steel $23\frac{5}{8}$ in. long is divided into 7 equal parts. What is the length of each part?

SOLUTION. The length of each part must be $23\frac{5}{8} \div 7$. Now $23\frac{5}{8} = \frac{189}{8}$ and 7 inverted is $\frac{1}{7}$. Then $23\frac{5}{8} \div 7 = \frac{189}{8} \div 7 = \frac{\overset{27}{\cancel{189}}}{8} \times \frac{1}{\cancel{7}} = \frac{27}{8} = 3\frac{3}{8}$. Ans.

Check Your Learning

76. In the division of fractions, the first step is to ________________ the divisor. (invert)

77. When dividing fractions, after the divisor has been inverted, proceed as in ________________ . (multiplication)

78. In a division problem the divisor always (precedes, follows) the division sign. (follows)

79. The inverted form of the fraction $\frac{2}{3}$ is __________ . ($\frac{3}{2}$)

Practice Problems

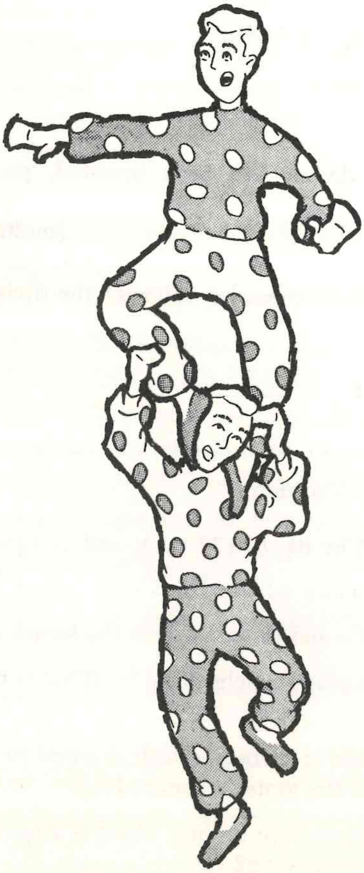
1. Perform the following divisions: *a)* 15 by $6\frac{3}{7}$, *b)* 172 by $\frac{4}{5}$, and *c)* $\frac{103}{6}$ by $14\frac{2}{3}$.
2. How many gears, each $1\frac{1}{8}$ in. thick, can be set side by side in a space 45 in. long?
3. A bar $34\frac{7}{32}$ in. long is divided into 15 equal parts. What is the length of each part?
4. A boiler plate containing 24 sq ft of surface weighs $362\frac{2}{5}$ lb. What is its weight per square foot?
5. A certain boiler has $927\frac{1}{2}$ sq ft of heating surface, which is equal to 35 times the area of the grate. What is the area of the grate in square feet?
6. If the distance around the rim of a locomotive driving wheel is $13\frac{1}{12}$ ft, how many revolutions will the wheel make in traveling 682 ft?

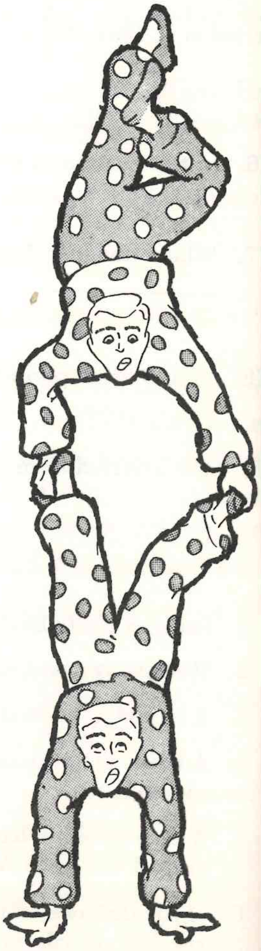
7. A tank containing 1830 gal of water supplies a family whose daily consumptio according to a meter, averages $521\frac{1}{2}$ gal. How many days' supply does the ta contain?

8. If 1 hp will drive $3\frac{3}{4}$ cotton looms, how much power will be required for a wea room containing 600 of these looms?

9. How many bolts can be obtained from a steel bar $15\frac{3}{4}$ in. in length, if each bo requires a piece $3\frac{15}{16}$ in. long?

10. A piece of land having an area of $12\frac{3}{4}$ acres is to be divided into plots, each pl containing $\frac{3}{8}$ of an acre. How many plots will there be?





Reciprocals

42. No text on fractions would be complete without a discussion of reciprocals. Just what the reciprocal of a number is and how to find it are things you'll need to know for later use.

The reciprocal of a number is the quotient of 1 divided by the number. Thus the reciprocal of 3 is $1 \div 3 = \frac{1}{3}$, and the reciprocal of 15 is $1 \div 15 = \frac{1}{15}$. The reciprocals of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{11}{12}$ are $1 \div \frac{1}{2} = 2$, $1 \div \frac{2}{3} = \frac{3}{2}$, and $1 \div \frac{11}{12} = \frac{12}{11}$. Notice that you can get the reciprocal of a fraction merely by writing the fraction upside down, that is, by inverting it. Thus the reciprocal of $\frac{1}{12}$ is $\frac{12}{1}$, or 12; that of $\frac{3}{4}$ is $\frac{4}{3}$; that of $\frac{11}{16}$ is $\frac{16}{11}$; and so on.

To get the reciprocal of a mixed number, you must first change the mixed number to an improper fraction. Then the reciprocal is easily obtained by inverting the improper fraction. For example, the reciprocal of $1\frac{3}{7}$ is found like this: $1\frac{3}{7} = \frac{10}{7}$. Inverting $\frac{10}{7}$, we get $\frac{7}{10}$. This is the reciprocal of $1\frac{3}{7}$. You can verify this by actually dividing $1\frac{3}{7}$ into 1.

Check Your Learning

80. The reciprocal of any number is equal to __________ divided by the number.

(1)

81. The reciprocal of 6 is $1 \div 6$, or __________.

($\frac{1}{6}$)

82. The reciprocal of a mixed number is a __________ __________.

(proper fraction)

83. The reciprocal of the fraction $\frac{6}{13}$ is __________.

($\frac{13}{6}$)

Practice Problems

Find the reciprocal of each of the following fractions:

1. $\frac{3}{4}$
2. $1\frac{1}{4}$
3. $\frac{5}{7}$
4. 8
5. 10
6. $\frac{11}{16}$
7. $5\frac{3}{4}$
8. $3\frac{3}{5}$
9. $7\frac{1}{8}$
10. $3\frac{1}{3}$

Complicated Fractions

43. The problems you'll meet in practice aren't always as simple as the ones you've met and solved here. For instance, the numerator and the denominator of a fraction may consist of fractions or mixed numbers. Such fractions could look like this:

$$\frac{\frac{1}{2}}{4} \qquad \frac{\frac{2}{3}}{\frac{3}{5}} \qquad \frac{6}{2\frac{7}{8}} \qquad \frac{1\frac{5}{7}}{\frac{2}{3}}$$

In dealing with these more complex types of fractions, you'll avoid confusio and difficulty if you keep one fact in mind: *In every fraction, the entire numerato must be divided by the entire denominator.* Think of the line of a fraction as "grouping symbol" which ties together all the numbers in the numerator and al the numbers in the denominator. Then take the fraction, no matter how compli cated it may seem, and rewrite it this way:

(the numerator) ÷ (the denominator)

When you rewrite even the most difficult fraction in this form, you'll be surprisec at how much simpler your work will be.

Now let's go back and solve the fractions we just mentioned.

$$\frac{\frac{1}{2}}{4} = (\tfrac{1}{2}) \div (4) = \tfrac{1}{2} \div \tfrac{4}{1} = \tfrac{1}{2} \times \tfrac{1}{4} = \tfrac{1}{8}$$

$$\frac{\frac{2}{3}}{\frac{3}{5}} = (\tfrac{2}{3}) \div (\tfrac{3}{5}) = \tfrac{2}{3} \times \tfrac{5}{3} = \tfrac{10}{9} = 1\tfrac{1}{9}$$

$$\frac{1\frac{5}{7}}{\frac{2}{3}} = (1\tfrac{5}{7}) \div (\tfrac{2}{3}) = \tfrac{12}{7} \div \tfrac{2}{3} = \frac{\overset{6}{\cancel{12}}}{7} \times \frac{3}{\cancel{2}} = \tfrac{18}{7} = 2\tfrac{4}{7}$$

$$\frac{6}{2\frac{7}{8}} = (6) \div (2\tfrac{7}{8}) = 6 \div \tfrac{23}{8} = 6 \times \tfrac{8}{23} = \tfrac{48}{23} = 2\tfrac{2}{23}$$

Notice that in these examples it wasn't absolutely necessary to use the parentheses, because each numerator and denominator consisted of a single number. However, we used the parentheses here so that you could get used to the proper procedure in a simple situation. Naturally, you'll have to use parentheses in the more complex problems, and so it's a good idea to make a habit of using them even when they're not essential.

More About Fractions

More Complicated Fractions

44. You've just seen how fractions can become quite complex when their terms involve other fractions. But the terms of fractions can become even more complicated. For example, the numerator and the denominator of a fraction may contain expressions that involve addition, subtraction, multiplication, or division. Here too just remember that in every fraction you must divide the numerator by the denominator. So how do we go about finding the value of a fraction when addition, subtraction, multiplication, and division are involved? This brings us to a principle called the *order of operations.* Stated simply, this principle goes like this: *You must multiply and divide before you add and subtract.* Let's apply this to an example: $\frac{\frac{1}{4}+\frac{2}{3}\times 9}{7\frac{1}{2}-3\frac{1}{2}}$. Now you know from your rule that you must not add $\frac{1}{4}$ to $\frac{2}{3}$ and then multiply the result by 9. Instead, you must multiply $\frac{2}{3}$ by 9 first, and then add $\frac{1}{4}$ to the product. This gives you $6\frac{1}{4}$ in your numerator. Then subtract $3\frac{1}{2}$ from $7\frac{1}{2}$ and you will get 4, which is your denominator. Now you're ready to divide $6\frac{1}{4}$ by 4, which gives you the correct answer, $\frac{25}{16}$, which can be reduced to $1\frac{9}{16}$. Your work will look like this:

$$\frac{\frac{1}{4}+\frac{2}{3}\times 9}{7\frac{1}{2}-3\frac{1}{2}}=(\tfrac{1}{4}+\tfrac{2}{3}\times 9)\div(7\tfrac{1}{2}-3\tfrac{1}{2})=(\tfrac{1}{4}+6)\div 4=\tfrac{25}{4}\times\tfrac{1}{4}=\tfrac{25}{16}=1\tfrac{9}{16}$$

Here are a few more examples and their solutions to show you how to apply the principles you've just learned.

EXAMPLE 1. Find the value of $\frac{\frac{1}{4}+\frac{2}{3}\times 12}{10\frac{15}{16}-\frac{3}{4}\times 12}$.

SOLUTION. Writing the fraction in the recommended form, we get

$$(\tfrac{1}{4}+\tfrac{2}{3}\times 12)\div(10\tfrac{15}{16}-\tfrac{3}{4}\times 12)$$

First, we'll simplify the numbers in the first parentheses.

$$\tfrac{1}{4}+\tfrac{2}{3}\times 12=\tfrac{1}{4}+8=8\tfrac{1}{4}$$

Then the second parentheses:

$$10\tfrac{15}{16}-\tfrac{3}{4}\times 12=10\tfrac{15}{16}-9=1\tfrac{15}{16}$$

Now our expression becomes

$$8\tfrac{1}{4}\div 1\tfrac{15}{16}=\tfrac{33}{4}\div\tfrac{31}{16}=\tfrac{33}{4}\times\tfrac{16}{31}=\tfrac{132}{31}=4\tfrac{8}{31}.\quad\text{Ans.}$$

EXAMPLE 2. Find the value of $\frac{1}{\frac{1}{4}+\frac{1}{5}+\frac{1}{7}+\frac{1}{10}}$.

SOLUTION. Here we get $(1)\div(\frac{1}{4}+\frac{1}{5}+\frac{1}{7}+\frac{1}{10})$. To reduce the fractions in the second parentheses to a single fraction, we proceed like this (note that the least common denominator is 140):

$$\tfrac{1}{4}+\tfrac{1}{5}+\tfrac{1}{7}+\tfrac{1}{10}=\tfrac{35}{140}+\tfrac{28}{140}+\tfrac{20}{140}+\tfrac{14}{140}=\tfrac{97}{140}$$

Now we have

$$1 \div \frac{97}{140} = 1 \times \frac{140}{97} = \frac{140}{97} = 1\frac{43}{97}. \quad \text{Ans.}$$

Now, let's take a look at some examples worked in less detail. See if you ca follow them.

EXAMPLE 3. Find the value of $\dfrac{2}{\frac{1}{4}+\frac{1}{6}}$.

SOLUTION. $\dfrac{2}{\frac{1}{4}+\frac{1}{6}} = (2) \div (\frac{1}{4}+\frac{1}{6}) = 2 \div \frac{5}{12} = 2 \times \frac{12}{5} = \frac{24}{5} = 4\frac{4}{5}.$ Ans.

EXAMPLE 4. Find the value of $\dfrac{10 \div 5 + 2}{6 + 3 \times \frac{2}{3}}$.

SOLUTION. $\dfrac{10 \div 5 + 2}{6 + 3 \times \frac{2}{3}} = (10 \div 5 + 2) \div (6 + 3 \times \frac{2}{3}) = 4 \div 8 = \frac{1}{2}.$ Ans.

EXAMPLE 5. Find the value of $\dfrac{3\frac{1}{2} \div 7 + 2}{\frac{1}{2} + 2 \times \frac{3}{5}}$.

SOLUTION.
$$\frac{3\frac{1}{2} \div 7 + 2}{\frac{1}{2} + 2 \times \frac{3}{5}} = (3\tfrac{1}{2} \div 7 + 2) \div (\tfrac{1}{2} + 2 \times \tfrac{3}{5})$$
$$= (\tfrac{7}{2} \times \tfrac{1}{7} + 2) \div (\tfrac{1}{2} + \tfrac{6}{5}) = 2\tfrac{1}{2} \div \tfrac{17}{10} = \tfrac{25}{17} = 1\tfrac{8}{17}. \quad \text{Ans.}$$

Check Your Learning

84. In every fraction, the entire ________________ must be divided by the entire ________________.

(numerator) (denominator)

85. The rule which states that you must multiply and divide before you add and subtract is called the order of ________________.

(operations)

86. The expression $\dfrac{(3 + 6 \times 8 - 2)}{25 + 25}$ is equal to ____________.

($\frac{49}{50}$)

87. If there are any points not perfectly clear to you, be sure to ________________ the text explanation right now.

(review)

Practice Problems

Now test your ability to handle the more complex fractions.

1. $\dfrac{\frac{7}{10}}{\frac{2}{3}}$

2. $\dfrac{1\frac{7}{8}}{\frac{15}{16}}$

3. $\dfrac{3}{3\frac{1}{4}}$

4. $\dfrac{1 + 3 - 5 \div 2}{\frac{2}{3} \times \frac{1}{4} + 1}$

5. $\dfrac{7+1\times 2-\frac{3}{2}}{5-2+3-\frac{1}{3}\times 12}$

6. $\dfrac{2}{\frac{1}{3}+\frac{1}{8}+\frac{1}{12}}$

7. $4+\dfrac{1}{\frac{1}{9}+\frac{1}{18}+\frac{1}{36}+\frac{1}{72}}$

8. $\dfrac{1}{\frac{1}{4}+\frac{1}{6}+\frac{1}{12}+\frac{1}{18}}$

9. $\dfrac{10\div\frac{2}{3}-6\times\frac{4}{5}}{6\frac{4}{5}+8\frac{1}{2}}$

10. $\dfrac{2\frac{2}{3}+14\div 10\frac{1}{2}}{4\frac{2}{3}\div 3\frac{1}{9}}$

Stop, Think, and Review

45. Now that you've reached this point, you may think that your study of fractions is finished. But actually, it has just begun! There are undoubtedly one or two sections of this text, maybe more, that you should restudy, and probably a number of points that aren't quite clear in your mind. For your own sake, don't ignore any unclear points. Remember that until you have a good understanding of everything discussed in this text, you aren't ready to answer the examination questions. And before you start to work on the examination, go back to your text *Whole Numbers* and review the hints given there for improving your work.

Answers to Practice Problems

Article 18

1. $\frac{56}{128}$
2. $\frac{2}{11}$
3. $\frac{8}{125}$
4. $\frac{35}{49}$
5. $\frac{650}{1000}$
6. $\frac{24}{64}$
7. $\frac{4500}{10000}$
8. $\frac{1}{13}$
9. $\frac{1}{11}$
10. $\frac{96}{144}$

Article 20

1. $\frac{131}{4}$
2. $\frac{113}{6}$
3. $\frac{89}{21}$
4. $\frac{343}{8}$
5. $\frac{80}{3}$
6. $\frac{53}{10}$
7. $\frac{217}{17}$
8. $\frac{365}{39}$
9. $\frac{145}{21}$
10. $\frac{193}{11}$

Article 21

1. $16\frac{7}{8}$
2. $71\frac{1}{3}$
3. $123\frac{1}{3}$
4. 55
5. 64
6. $61\frac{2}{3}$
7. $9\frac{3}{17}$
8. $20\frac{3}{13}$
9. $19\frac{7}{9}$
10. $12\frac{1}{11}$

Article 24

1. 24

2. 60

3. 72

4. 140

Article 25

1. $\frac{6}{8}, \frac{5}{8}, \frac{7}{8}$

2. $\frac{6}{32}, \frac{24}{32}, \frac{7}{32}$

3. $\frac{77}{88}, \frac{7}{88}, \frac{80}{88}$

4. $\frac{24}{40}, \frac{25}{40}, \frac{11}{40}$

5. $\frac{16}{40}, \frac{7}{40}, \frac{18}{40}$

6. $\frac{14}{30}, \frac{17}{30}, \frac{21}{30}$

7. $\frac{216}{612}, \frac{544}{612}, \frac{153}{612}$

8. $\frac{63}{90}, \frac{75}{90}, \frac{40}{90}$

9. $\frac{204}{255}, \frac{15}{255}, \frac{102}{255}$

10. $\frac{42}{54}, \frac{3}{54}, \frac{6}{54}$

Article 28

1. *a)* $1\frac{7}{12}$

b) $1\frac{8}{15}$

2. $7\frac{3}{32}$ in.

3. $4\frac{1}{2}$ hr

4. 1847 lb

5. 260 yd

6. 26 in.

7. 111 ft

8. 193 miles

9. $24\frac{3}{8}$ lb

10. 8 in.

Article 32

1. *a)* $\frac{11}{30}$

b) $17\frac{1}{4}$

c) $14\frac{7}{8}$

2. $8\frac{19}{32}$ in.

3. $\frac{5}{32}$ in.

4. $44\frac{1}{3}$ gal

5. $14\frac{19}{24}$ hp

6. $259\frac{2}{3}$ tons

7. $2\frac{15}{16}$ lb

8. $15\frac{11}{24}$ lb

9. $124\frac{7}{20}$ miles

10. $21\frac{7}{12}$ hp

Article 40

1. *a)* $\frac{36}{13}$, or $2\frac{10}{13}$

b) $\frac{40}{16}$, or $2\frac{1}{2}$

c) $\frac{22}{27}$

d) $\frac{19}{60}$

e) $\frac{3}{16}$

f) $\frac{1}{2}$

2. $15\frac{1}{2}$ in.

3. $33\frac{1}{3}$ in.

4. $153\frac{17}{21}$ hp

5. $796\frac{19}{20}$ lb

6. $1276\frac{1}{8}$ lb

7. $7299\frac{63}{100}$ gal

8. $5323\frac{1}{8}$ lb

Article 41

1. *a)* $2\frac{1}{3}$
   b) 215
   c) $1\frac{15}{88}$
2. 40 gears
3. $2\frac{9}{32}$ in.
4. $15\frac{1}{10}$ lb per sq ft
5. $26\frac{1}{2}$ sq ft
6. $52\frac{20}{157}$ revolutions
7. $3\frac{531}{1043}$ days
8. 160 hp
9. 4 bolts
10. 34 plots

Article 42

1. $\frac{4}{3}$
2. $\frac{4}{5}$
3. $\frac{7}{5}$
4. $\frac{1}{8}$
5. $\frac{1}{10}$
6. $\frac{16}{11}$
7. $\frac{4}{23}$
8. $\frac{5}{18}$
9. $\frac{8}{57}$
10. $\frac{3}{10}$

Article 44

1. $\frac{21}{20}$, or $1\frac{1}{20}$
2. 2
3. $\frac{12}{13}$
4. $1\frac{2}{7}$
5. $3\frac{3}{4}$
6. $3\frac{9}{13}$
7. $8\frac{4}{5}$
8. $1\frac{4}{5}$
9. $\frac{2}{3}$
10. $2\frac{2}{3}$

Practical Arithmetic

Serial 6750B

PART 2

Edition 1

Examination Questions

NOTICE TO STUDENTS: Study this instruction text thoroughly before you try to answer the following questions. Read each question carefully and be sure you understand it. Then write the best answer that you can. If your answer requires a mathematical solution, show enough of your work to indicate the method you used. Unless you do this we will have no way of knowing how you obtained your answer and your work will not be considered complete. After you have answered all the questions, double-check them to make sure that this is the best work you can do. Don't omit any questions.

Mail your work to us as soon as it is finished. DON'T HOLD IT until another examination is ready.

Note that the point value of each question is given in parentheses after the number of the question.

1. (10) Reduce each of the following to simplest form.

a) $\frac{9}{72}$ $\frac{1}{8}$

b) $\frac{21}{35}$ $\frac{3}{5}$

c) $4\frac{9}{6}$ $\frac{\cancel{9}^{3}}{\cancel{6}}$ $4\frac{3}{2} = 5\frac{1}{2}$

d) $18\frac{6}{16}$ $18\frac{3}{8}$

e) $23\frac{15}{5}$ $23\frac{15}{5} = 23\frac{3}{1}$
$= 26.$

2. (10) *a)* Reduce each of the following improper fractions to a mixed number.
$\frac{21}{5}$; $\frac{11}{4}$; $\frac{7}{6}$. $4\frac{1}{5}$; $2\frac{3}{4}$; $1\frac{1}{6}$.

b) Change the following mixed numbers to improper fractions.
$10\frac{5}{8}$; $4\frac{1}{4}$; $9\frac{11}{12}$. $\frac{85}{8}$; $\frac{17}{4}$; $\frac{119}{12}$.

$$\begin{array}{r} 12 \\ 9 \\ \hline 108 \\ \hline 119 \end{array}$$

3. (10) There are four fractions whose denominators are 15, 24, 45, and 18. Calculate the least common denominator of these fractions. Be sure to show all the work.

4. (10) Perform the addition or subtraction as indicated. (Show work.)

a) $1\frac{1}{2} + 6\frac{1}{3} + 2\frac{3}{4}$

b) $2\frac{2}{3} + 10\frac{1}{5} + 5\frac{6}{15}$

c) $12\frac{2}{3} - 7\frac{6}{7}$

d) $25 - 1\frac{11}{12}$

5. (10) Find the product of each of the following.

a) $3\frac{1}{3} \times 16\frac{3}{5}$ (Show work.)

b) $7 \times 6\frac{1}{8}$

6. (10) Find the quotient of each of the following. (Show work.)

a) $3\frac{2}{5} \div 2$

b) $22\frac{3}{16} \div 5\frac{5}{8}$

7. (4) Give the reciprocal for each of the following:

a) 4; *b)* $2\frac{1}{5}$; *c)* $\frac{5}{16}$; *d)* $\frac{7}{4}$.

$\frac{1}{4}$ $\frac{5}{11}$ $\frac{16}{5}$ $\frac{4}{7}$

WORK REQUIRED ON QUESTIONS 8, 9, AND 10.

8. (16) Find the value of each of the following expressions. Use cancellation i Part *c* and be sure to express your answer as a mixed number.

a) $\frac{1}{2} + \frac{4}{3 + 5}$

c) $\frac{17 \times 15 \times 8 \times 3}{24 \times 51 \times 2}$

b) $\frac{2}{3}$ of 36 divided by $\frac{1}{2}$

d) $\frac{13 + 8 + 9}{4 \times 2 - 3}$

9. (10) A family spends $\frac{1}{12}$ of its annual income of $16,620 for shelter, $\frac{1}{3}$ for food and clothing, $\frac{1}{6}$ for general expenses and $\frac{3}{20}$ for miscellaneous expenses.

a) What fractional part of the annual income is spent and what fractional part is left for savings? Express your answers as fractions in simplest form.

b) How much is spent for each item including savings?

10. (10) A man left an estate of $26,470. He gave $12,000 to his wife, $1000 to charity, and the remainder to his three children in equal shares. How much did each child receive?